نگار صمدانی

① $\frac{1}{2} = \frac{m-k}{4+2} \rightarrow \underline{m-k=-3}$

$\overline{AB} = \sqrt{(4+2)^2+(m-k)^2} \rightarrow \overline{AB} = \sqrt{36+9} = \sqrt{45}$

$m-k=-3$ $\qquad S = \overline{AB} \times \overline{AB} = 45$

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② $\overline{AB} = \sqrt{(1+3)^2+(4-1)^2} = 5$

$m\overline{AB} = \frac{4-1}{-1-3} = -\frac{3}{4} \rightarrow m_{BC} = \frac{4}{3}$ ✓

$m\overline{BC} = \frac{4}{3} = \frac{y-\cancel{4}}{x-4} \rightarrow \frac{4}{3} = \frac{y-\cancel{4}}{-\frac{3}{4}+\frac{1}{4}} \rightarrow y=1$

$\overline{CD} = 5 = \sqrt{(1+2x)^2+3^2} \rightarrow 4 = (1+2x) \rightarrow x = \frac{3}{2}$ ✓

$\rightarrow x = \frac{3}{2},\ y=1 \rightarrow C(\frac{3}{2}, 1)$

$\overline{BC} = \sqrt{(3-\frac{3}{2})^2+(3-1)^2} = \frac{5}{2}$

محیط $P = \frac{5}{2}+\frac{5}{2}+5+5 = 15 \rightarrow \boxed{P=15}$

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③ $y = -\frac{2m}{m^2-1} + \frac{3}{m^2-1} \rightarrow m = tg60° \Rightarrow \sqrt{3} = \frac{-2m}{m^2+1}$

$\sqrt{3}m^2+2m-\sqrt{3}=0 \rightarrow m = \frac{-2 \pm 4}{2\sqrt{3}}$

$m_1 = -\frac{3}{\sqrt{3}} = -3\sqrt{3}$

$m_2 = \frac{2}{2\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\rightarrow m_2-m_1=$

$m_2-m_1 = \frac{\sqrt{3}}{3}+3\sqrt{3} = 10\sqrt{3}$

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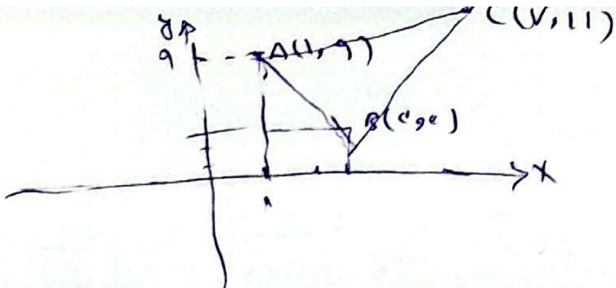

خط $\overline{BC}$: $m_{BC} = \dfrac{۱۱-۳}{۷-۳} = \dfrac{۸}{۴} = ۲ \rightarrow y = ۲x + H$ $(۳,۳)$ $\quad ۳ = ۶ + H \rightarrow H = -۳$

خط $\overline{BC}$: $y = ۲x - ۳ \rightarrow y - ۲x + ۳ = ۰$

$$\overline{AH} = \dfrac{|۹-۲+۳|}{\sqrt{۱+۲^۲}} = \dfrac{۱۰}{\sqrt{۵}} = ۲\sqrt{۵} \rightarrow AH = ۲\sqrt{۵}$$

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(۵)

$$\begin{matrix} AB \\ DC \end{matrix}\begin{cases} y + ۲x = ۷ \\ ۲y - ۷x = -۱۹ \end{cases} \rightarrow \begin{cases} -۲y - ۴x = -۱۴ \\ ۲y - ۷x = -۱۹ \end{cases} \rightarrow \begin{matrix} x = ۳ \\ y = ۱ \end{matrix} \rightarrow A(۳,۱)$$

$$BH = \dfrac{|۴\times۱ - ۳\times۳ - ۱۷|}{\sqrt{۱۶+۹}} = \dfrac{+۲۲}{۵} = ۴{,}۴$$

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$$m = \dfrac{-۶-۰}{۰+\frac{۳}{۴}} = -۸ \rightarrow y = -۸x - ۶$$

A نقطه (x,x)

$$x = -۸x - ۶ \rightarrow ۹x = -۶$$

$$x = \dfrac{-۶}{۹}$$

$$A\left(-\dfrac{۶}{۹}, -\dfrac{۶}{۹}\right)$$

$$\overline{AO} = \sqrt{\left(\dfrac{۶}{۹}\right)^۲ + \left(\dfrac{۶}{۹}\right)^۲} = \sqrt{۲}\times\dfrac{۶}{۹} = \dfrac{۶}{۹}\sqrt{۲}$$

$$\begin{matrix} y = ax + ۱ \\ y = \dfrac{x}{a} + \dfrac{a-۱}{a} \end{matrix} \rightarrow m = m' \rightarrow a = \dfrac{۱}{a} \rightarrow a = \pm ۱$$

$$\begin{cases} y = x + ۱ \\ y = x \end{cases} \rightarrow \dfrac{c-c'}{\sqrt{۲}} = \dfrac{۱}{\sqrt{۲}} \rightarrow \dfrac{\sqrt{۲}}{۲}$$

$$y = ax + 1$$
$$y = \frac{x}{a} + \frac{a-1}{a} \rightarrow m = m' \rightarrow a = \frac{1}{a} \rightarrow \boxed{a = ۱}$$

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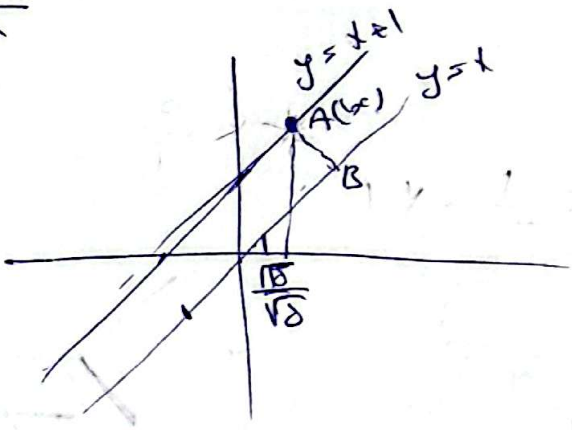
خطوط ما $\begin{cases} y = x + ۱ \\ y = x \end{cases}$



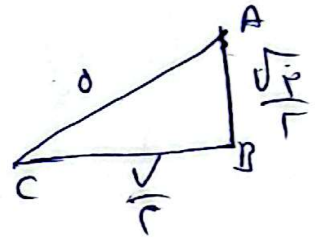
$$AC = ۵ = \sqrt{x^۲ + x^۲} \rightarrow ۵ = \sqrt{۲}\,x \rightarrow \left(x = \frac{۵}{\sqrt{۲}}\right)$$

نقطه C رو $y = x$

$$AB = \frac{|c - c'|}{\sqrt{۲}} = \frac{۱}{\sqrt{۲}} = \left(\frac{\sqrt{۲}}{۲}\right), \quad AB = \frac{\sqrt{۲}}{۲}$$

$$AC = ۵$$
$$BC = \sqrt{۲۵ - \frac{۱}{۲}} = \frac{۷}{۲}$$

$$S = \frac{۷}{۲} \times \frac{\sqrt{۲}}{۲} = \frac{۷\sqrt{۲}}{۴}$$



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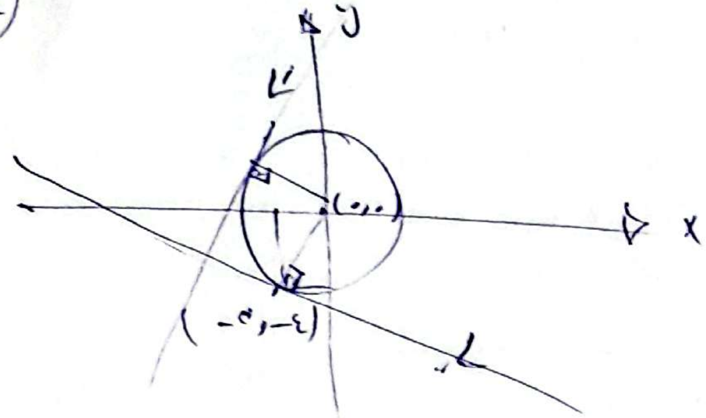

$$mL' = \frac{۴}{۳} \quad , \quad mL = -\frac{۳}{۴}$$

$$L: \; y = -\frac{۳}{۴}x + H$$

$$(-۳,-۴) \longrightarrow H = -۴ - \frac{۹}{۴} = -\frac{۲۵}{۴}$$

$$L \quad \boxed{y = -\frac{۳}{۴}x - \frac{۲۵}{۴}}$$

$$L' \quad y = \frac{۴}{۳}x + H$$

$$R = ۵ \longrightarrow \quad ۵ = \frac{|H|}{\sqrt{۱ + \frac{۱۶}{۹}}} = \frac{H}{\frac{۵}{۳}} = ۵$$

$$H = \frac{۲۵}{۳}$$

$$\boxed{L': \; y = \frac{۴}{۳}x + \frac{۲۵}{۳}}$$

$$\begin{cases} y = \frac{۴}{۳}x + \frac{۲۵}{۳} \\ y = -\frac{۳}{۴}x - \frac{۲۵}{۴} \end{cases} \longrightarrow \frac{۲۵}{۱۲}x = -\frac{۱۷۵}{۱۲} \longrightarrow \begin{cases} x = -۷ \\ y = -۱ \end{cases}$$

$$\longrightarrow \boxed{xy = ۷}$$

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$$\sqrt{۳} = \frac{b-a}{-\frac{۱}{۳} + \frac{۱}{۲}} = \frac{b-a}{+\frac{۱}{۶}} \rightarrow b - a = \frac{\sqrt{۳}}{۶}$$

$$AB = \sqrt{(b-a)^۲ + (\tfrac{۱}{۲} - \tfrac{۱}{۳})^۲} = \sqrt{\frac{۳}{۳۶} + \frac{۱}{۳۶}} = \frac{۲}{۶}$$

پس $AC = \sqrt{۲}\,AB = \frac{\sqrt{۲}}{۳}$