①

$y=(a-1)x^2+x+3$ متقارن $x=2$ $\quad x_0=?$

$x=\frac{-b}{2a}=2 \to \frac{-1}{2(a-1)}=2 \to -1=4(a-1) \to -1=4a-4$

$a=\frac{3}{4} \to y=-\frac{1}{4}x^2+x+3$

$x_0 \to 0=-\frac{1}{4}x^2+x+3 \to -4x^2+4x+12$ $\begin{cases} x=6 & \text{ق ق} \\ x=-2 & \text{غ ق} \end{cases}$

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②

$f(x)=mx^2-5x+m$ محدوده $m=?$

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

$\Delta>0 \; , \; m<0$

$b^2-4ac>0 \to 25-4m^2>0 \to m^2<\frac{25}{4}$

$-\frac{5}{2}<m<\frac{5}{2}$ if $m<0 \Rightarrow m=(-\frac{5}{2},0)$

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③

$x_1=\frac{3-\sqrt{5}}{3} \; , \; x_2=\frac{3+\sqrt{5}}{3}$

$P=(\frac{3-\sqrt{5}}{3})\times(\frac{3+\sqrt{5}}{3}) \Rightarrow \frac{4}{9}$ $\qquad S=\frac{3-\sqrt{5}}{3}+\frac{3+\sqrt{5}}{3}=2$

معادله $\to x^2-Sx+P=0 \to x^2-2x+\frac{4}{9} \to 9x^2-18x+4=0$

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④

$2x^2-8x+m+2=0 \qquad \alpha<\beta \; , \; 3\alpha^2+\beta^2=12 \qquad m=?$

$3\alpha^2+\beta^2=12 \to 2\alpha^2+\underbrace{(\alpha^2+\beta^2)}_{S^2-2P}=12$

$\left.\begin{array}{l} 2\alpha^2+16-m-2=12 \\ 2\alpha^2=8\alpha-m-2 \end{array}\right\} 8\alpha-2m+12=12 \to m=4\alpha$

$\alpha\beta=\frac{m+2}{2} \to \frac{4\alpha+2}{2}=2\alpha+1 \to \beta=\frac{2\alpha+1}{\alpha} \to \beta=3$

$\alpha+\beta=\frac{2\alpha+1}{\alpha}+\frac{\alpha^2}{\alpha}=\frac{\alpha^2+2\alpha+1}{\alpha}=4 \to \alpha=1 \to m=4$

احمد رهاش
تکلیف ۱۳

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$S(۲,۹) \quad y_0 = ۵ \quad x > 0 \ ?$

$\to y = a(x-۲)^۲ + ۹$
$y = a(x) + ۵$
$\} \ a = -۱$

$\to y = -x^۲ + ۴x + ۵$

$-x^۲ + ۴x + ۵ > 0$

x		-۱		۵	
y	$-$	$\vert$	$+$	$\vert$	$-$

$\rightsquigarrow (-۱, ۵)$

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$ax^۲ - ۷x + ۹\beta = 0 \qquad \beta > 0 \qquad \frac{1}{\alpha} + \frac{1}{\beta} = ?$

$\underset{\alpha+\beta}{S} = \frac{۷}{\alpha} \qquad \underset{\alpha\beta}{P} = \frac{۹\beta}{\alpha} \to ۹\beta = \alpha^۲\beta \to \alpha = \pm ۳$

if $\alpha = ۳ \to \alpha + \beta = \frac{۷}{\alpha} \to \beta = \frac{-۲}{۳}$ غ ق ق

if $\alpha = -۳ \to \alpha + \beta = \frac{-۷}{\alpha} \to \beta = \frac{۲}{۳}$ ق ق

$\frac{1}{\alpha} + \frac{1}{\beta} = -\frac{۱}{۳} + \frac{۳}{۲} = \frac{۷}{۶}$

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$x^۲ + mx - ۲m = 0 \qquad \alpha^۲ - m\beta = ۸ \qquad \overset{\alpha}{x_1} + \overset{\beta}{x_2} = ?$

$S = -m \qquad P = -۲m$

$\beta = \frac{-۲m}{\alpha} \to \alpha^۲ + \frac{۲m^۲}{\alpha} = ۸ \to \alpha^۳ + ۲m^۲ - ۸\alpha = 0$

$\to \alpha + -\frac{۲m}{\alpha} = -m \to m = \frac{\alpha^۲}{۲-\alpha}$

m جایگذاری $\to \alpha^۳ + ۲\left(\frac{\alpha^۲}{۲-\alpha}\right)^۲ - ۸\alpha = 0 \to \alpha \simeq ۱/۲$ $\} \ m \simeq ۲$

$\alpha + \beta = -m \rightsquigarrow \alpha + \beta \simeq -۲$

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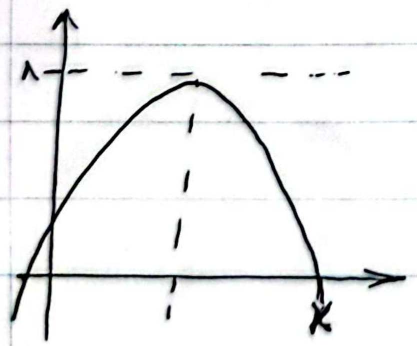
$f(x) = mx^۲ + ۴x + \frac{m}{۲} + ۷ \to$ تابع رو به پایین $\to m < 0$



$-\frac{\Delta}{۴a} = ۸ \to -\frac{b^۲ + ۴ac}{۴a} = ۸$

$\to \frac{-۱۶ + ۴m(\frac{m}{۲} + ۷)}{۴m} = ۸$

$\to m = -۲$ ق ق

$m = ۴$ غ ق ق

$f(x) = -۲x^۲ - ۸x + ۶$

$0 = +x^۲ + ۴x - ۳ \to \Delta = ۲۸$

$x_1 = -۲ \pm \sqrt{۷}$

$-۲ + \sqrt{۷}$ ق ق

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راطمره رزنی
صنف ۱۳

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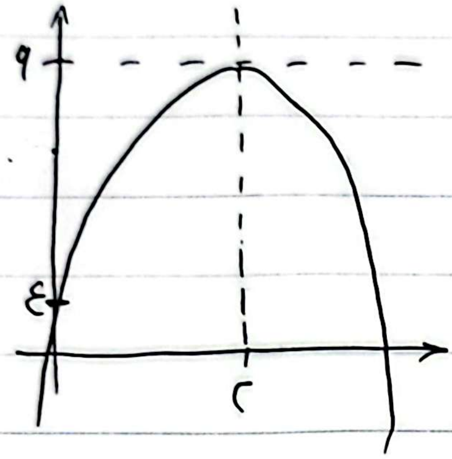

$y = a(x-h)^2 + k = a(x-2)^2 + 6$

$4 = a(4) + 6 \quad \Rightarrow \quad a = -\frac{1}{2}$

$y = -\frac{1}{2}x^2 + 2x + 4$

$P = -8 \qquad S = 4$

مجموع معکوس ریشه ها $\Rightarrow \frac{1}{x_1} + \frac{1}{x_2} \rightarrow \frac{x_1 + x_2}{x_1 x_2} = \frac{S}{P}$

$= -\frac{1}{2}$

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$x^2 - (4a+4)x + (2a^2 + 6a + 2) = 0 \qquad \alpha = 2 \quad \beta = ?$

$\hookrightarrow 4 - 8a - 8 + 3a^2 + 6a + 2) = 0$

$3a^2 - 2a - 1 = 0 \rightarrow a = 1 \rightarrow x^2 - 8x + 12 \rightarrow x = 2,\ x = 6$

$\hookrightarrow a = -\frac{1}{3} \rightarrow x^2 - \frac{8}{3}x + \frac{4}{3} \rightarrow x = \frac{2}{3},\ x = 2$

اگر $a = 1 \rightarrow \beta = 6$

اگر $a = -\frac{1}{3} \rightarrow \beta = \frac{2}{3}$

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