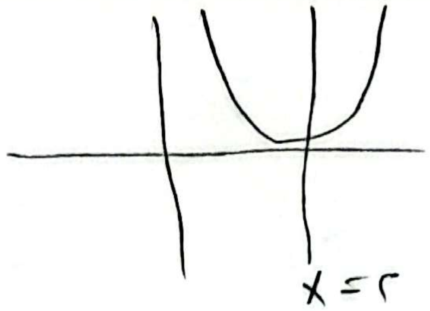
①



$\rightarrow x_S = 2 \rightarrow \frac{-1}{2a-2} = 2 \rightarrow 4a-4=-1$

$a=\frac{3}{4}$

$y = -\frac{1}{4}x^2 + x + 3$

$-x^2+4x+12=0$

$(x-6)(x+2)=0$ / $x=-2$, $x=6$

$\boxed{x=6}$

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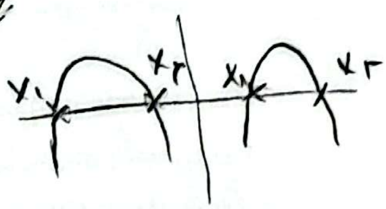
② $f(x) = mx^2 - 5x + m$

x		x_1		x_2	
$f(x)$	$-$	0	$+$	0	$-$

$a<0$



$\frac{c}{a} = \frac{m}{m} = 1 \rightarrow \frac{c}{a} > 0 \rightarrow$ دو ریشه هم علامت

① $\begin{cases} a<0 \rightarrow c<0 \\ m<0 \end{cases}$

② $\Delta > 0 \rightarrow 25 - 4m^2 \geq 0 \quad -\frac{5}{2} \leq m \leq \frac{5}{2}$

$\boxed{-\frac{5}{2} \leq m < 0}$

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③ $\alpha = \frac{3-\sqrt{5}}{2}$, $\beta = \frac{3+\sqrt{5}}{2}$

$\alpha\beta = \frac{9-5}{9} = \frac{4}{9} \rightarrow$ $x^2 - Sx + P = 0$

$\alpha + \beta = 2$

$\boxed{x^2 - 2x + \frac{4}{9} = 0}$

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④ $2x^2 - 8x + m + 2 = 0 \quad \alpha < \beta$

$3\alpha^2 + \beta^2 = 12$

$\alpha + \beta = 4$

$\alpha\beta = \frac{m+2}{2}$

$2\alpha^2 - 8\alpha + m + 2 = 0$

$+\ 2\beta^2 - 8\beta + m + 2 = 0$

$\rightarrow 2(\alpha^2+\beta^2) - 8(\alpha+\beta) + 2m + 4 = 0$

$2((\alpha+\beta)^2 - 2\alpha\beta) - 8(\alpha+\beta) + 2m + 4 = 0$

$2(16 - m - 2) - 8 \times 4 = 2m + 4$

$32 - 2m - 4 - 32 = 2m + 4 \rightarrow 4m = -8 \rightarrow \boxed{m=-2}$

⑥

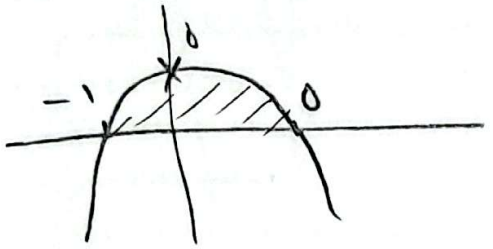
$y = a(x-4)^2 + 4$

$y = a(x-2)^2 + 9$

$x = 0 \rightarrow 0 = 4a + 9 \rightarrow \boxed{a = -1}$

$y = -(x-2)^2 + 9 \rightarrow y = -x^2 + 4x + 5$

$-x^2 + 4x + 5 = 0 \rightarrow (x-5)(x+1)$



$-1 \leq x \leq 5$

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④ $2x^2 - 7x + 6 = 0$

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha+\beta}{\alpha\beta} = \frac{\frac{7}{2}}{3} = \frac{7}{6} \rightarrow \boxed{\frac{1}{\alpha} + \frac{1}{\beta} = \frac{7}{6}}$

$\alpha + \beta = \frac{7}{2}$

$\alpha\beta = \frac{6}{2} \rightarrow \alpha^2\beta = 6\alpha \rightarrow \alpha = 3$ ✓

$\alpha = 3 \quad 9 + \beta = \frac{7}{2} \rightarrow \beta = \frac{1}{2}$ ✓ $\rightarrow \alpha = 3, \beta = \frac{1}{2}$

$\alpha = 2 \rightarrow \alpha + \beta = \frac{7}{2} \rightarrow \beta = \frac{3}{2}$ ✗

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⑦ $x^2 + mx - 2m = 0 \quad \alpha + \beta = ? \quad \alpha + \beta = -m$

$\alpha^2 - m\beta = 7$

$\alpha^2 + m\alpha - 2m = 0 \rightarrow \alpha^2 = 2m - m\alpha$

$2m - m\alpha - m\beta = 7$

$\begin{cases} 2m - (\alpha+\beta)m = 7 \\ \alpha + \beta = -m \end{cases} \rightarrow 2m + m^2 + 7 = 0$

$m^2 + 2m - 7 = 0$

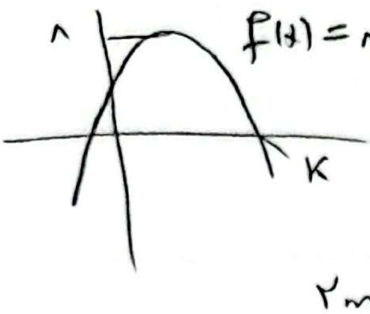
$(m+4)(m-2) = 0$

$m = -4 \rightarrow x^2 - 4x + 7 = 0 \quad \Delta < 0$

$m = 2 \quad x^2 + 2x - 4 = 0 \quad \Delta > 0$

$m = 2 \rightarrow -\frac{b}{a} = \frac{-2}{1} = -2 \rightarrow \boxed{\alpha + \beta = -2}$

(٨)



$f(x) = mx^٢ + ٤x + \frac{m}{٢} + ٧$

$y_S = \frac{-\Delta}{٤a} \Rightarrow ٨ = -\frac{(١٦ - ٢m^٢ - ٢٨m)}{٤m} = ٨$

$٢m^٢ + ٢٨m - ١٦ = ٣٢m$

$٢m^٢ - ٤m - ١٦ = ٠ < \begin{matrix} m = -٢ \\ m = ٤ \end{matrix}$

$f(x) = -٢x^٢ + ٤x + ٦ \longrightarrow x = ٣ , \; x = -١ \longrightarrow$ $\boxed{K = ٣}$

$m = -٢$

$f(x) = ٤x^٢ + ٤x + ٩ \quad \Delta < ٠$

$m = ٤$

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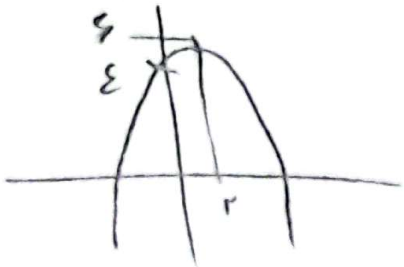
(٩)



$y = a(x - x_S)^٢ + y_S$

$y = a(x - ٢)^٢ + ٦$

$x = ٠ \rightarrow y = ٤ \longrightarrow ٤a + ٦ = ٤ \rightarrow a = -\frac{١}{٢}$

$y = -\frac{١}{٢}(x - ٢)^٢ + ٦$

$y = -\frac{١}{٢}x^٢ + x - ٢ + ٦$

$y = -\frac{١}{٢}x^٢ + x + ٤ \rightarrow \frac{١}{\alpha} + \frac{١}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{٢}{-٨} = -\frac{١}{٤}$

$\boxed{\frac{١}{\alpha} + \frac{١}{\beta} = -\frac{١}{٤}}$

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(١٠) $x^٢ - (٤a + ٤)x + ٣a^٢ + ٦a + ٥ = ٠$

$x = ٢ \quad ٤ - ٨a - ٨ + ٣a^٢ + ٦a + ٥ = ٠$

$٣a^٢ - ٢a - ١ = ٠ \longrightarrow \frac{٢ \pm ٤}{٢ \times ٣} < \begin{matrix} a = ١ \\ a = -\frac{١}{٣} \end{matrix}$

$a = ١ \rightarrow x^٢ - ٨x + ١٢ = ٠ \rightarrow (x - ٢)(x - ٦) = ٠ \rightarrow x = ٦$

$a = -\frac{١}{٣} \rightarrow x^٢ - \frac{٨}{٣}x + \frac{٤}{٣} = ٠ \quad x = ٢ \rightarrow , \; x = \frac{٢}{٣} \longrightarrow$ $\boxed{\begin{matrix} x = ٦ \\ x = \frac{٢}{٣} \end{matrix}}$