(۱۹)

مائده میرمقتدایی

۱ ←

$-\frac{b}{2a} = 2 \to \frac{-1}{a-1} = 4 \to -1 = 4a - 4 \to a = \frac{3}{4}$

(۳)

$\to -\frac{1}{4}x^2 + x + 3 = 0 \xrightarrow{\times 4} -x^2 + 4x + 12 = 0 \xrightarrow{\times -1} x^2 - 4x - 12 = 0$

$\Rightarrow (x-6)(x+2) \to x = -2, 6 \xrightarrow{\text{طول}} \boxed{x = 6}$

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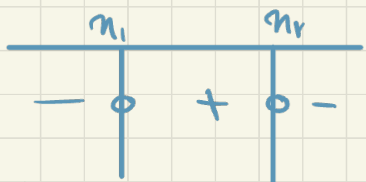
۲ ←

$f(x) = mx^2 - 5x + m$



با توجه به جدول تعیین علامت $\to m > 0$ $\quad m < 0$

$\Delta > 0 \to 25 - 4m^2 > 0 \to 2m < 5 \to m < \frac{5}{2}$

$\Rightarrow \boxed{m \in (0, \frac{5}{2})}$

$-\frac{5}{2} < m < 0$

(۱)

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۳ ←

$\frac{3-\sqrt{5}}{3}, \frac{3+\sqrt{5}}{3} \to ax^2 - Sx + P$

$S = \frac{3-\sqrt{5}+3+\sqrt{5}}{3} = 2$

$P = \frac{(3-\sqrt{5})(3+\sqrt{5})}{3 \times 3} = \frac{9-5}{9} = \frac{4}{9}$

$\to x^2 - 2x + \frac{4}{9} = 0$

$\Rightarrow \boxed{9x^2 - 18x + 4 = 0}$

(۳)

$\leftarrow 4$

$\alpha+\beta=-\frac{b}{a}=4 \quad , \quad \alpha\beta=\frac{c}{a}=\frac{m+2}{2}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\beta=4-\alpha \longrightarrow 3\alpha^2+(4-\alpha)^2=12 \longrightarrow$

3)

$4\alpha^2-8\alpha+4=0 \longrightarrow \alpha^2-2\alpha+1=0 \longrightarrow (\alpha-1)^2=0$

$\longrightarrow \alpha=1 \longrightarrow \beta=4-1=3 \longrightarrow \alpha\beta=3 \Longrightarrow$

$6=m+2 \longrightarrow \boxed{m=4}$

$y=a(x-h)^2+k \longrightarrow y=a(x-2)^2+9 \longrightarrow$ $\leftarrow 5$

$(x=0 \rightarrow y=5)$

$5=4a+9 \longrightarrow a=-1 \Longrightarrow y=-1(x-2)^2+9$ $\left.\right\} \longrightarrow$

$(x-2)^2 = x^2+4-4x$

$y=-x^2+4x+5 \longrightarrow -(x-5)(x+1) \longrightarrow$

4)

	-1		5	
$-$	0	$+$	0	$-$

$\Longrightarrow$

$\boxed{(-1,5)}$

$\alpha x^2 - 7x + 9\beta = 0$ ← 6

$\beta > 0 ,\quad \dfrac{\alpha+\beta}{\alpha\beta} = ?$

$\alpha + \beta = \dfrac{7}{\alpha} ,\quad \alpha\beta = \dfrac{9\beta}{\alpha} \to \alpha^2 = 9 \to \alpha = \pm 3$

$\xrightarrow{\alpha=-3} 9 - 3\beta = 7 \to -3\beta = -2 \to \beta = \frac{2}{3}$ ✓

$\xrightarrow{\alpha=3} 9 + 3\beta = 7 \to \beta = -\frac{2}{3}$ غ ق ق ✗

$$\frac{-3+\frac{2}{3}}{-3\times\frac{2}{3}} = \frac{\frac{-9+2}{3}}{-2} = \frac{\frac{-7}{3}}{-2} = \boxed{\frac{7}{6}}$$

$\alpha,\beta \to x^2 + mx - 2m = 0 \xrightarrow{x=\beta} \beta^2 + m\beta - 2m = 0 \to$ ← 7

$m\beta = 2m - \beta^2$
$\alpha^2 - m\beta = 8$
$\to \underbrace{\alpha^2+\beta^2}_{S^2-2P} - 2m = 8 \to$

$\alpha + \beta = ?$

$\frac{c}{a} = \alpha\beta = -2m ,\quad \alpha + \beta = -m$

$(\alpha+\beta)^2 + \cancel{2m}\ 4m - 2m = 8$

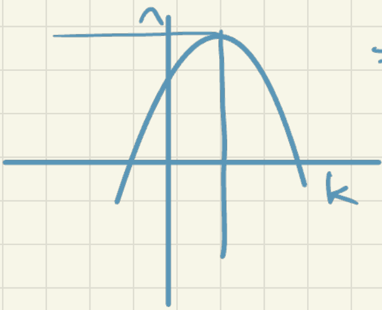
$\underbrace{(\alpha+\beta)^2}_{-m} + 2m = 8 \to m^2 + 2m - 8 = 0 \to (m-2)(m+4) = 0$

$m = 2 ,\quad m = -4$

$x^۲+۲x-۴=0 \rightarrow \Delta=۴+۴\times۴\times۱>0$ ✓

$x^۲+۴x+۸=0 \rightarrow \Delta=۱۶-۴\times۸\times۱<0$ ✗

$\left.\begin{array}{l}\alpha+\beta=-m \\ m=۲\end{array}\right\} \rightarrow \boxed{\alpha+\beta=-۲}$



$f_{max}=f(-\frac{b}{۲a})=f(-\frac{۲}{m})=۸$ ← ۸

$f(-\frac{۲}{m})=m(\frac{۴}{m^۲})+۴(-\frac{۲}{m})+\frac{m}{۲}+۷$

$\Rightarrow -\frac{۴}{m}+\frac{m}{۲}+۷=۸ \rightarrow m^۲-۲m-۸=0$

با توجه به نمودار $\rightarrow m<0 \rightarrow m=-۲ \rightarrow$

$-۲x^۲+۴x+۶ \rightarrow x=۳,-۱ \rightarrow \boxed{k=۳}$

$-\frac{b}{۲a}=۲ \quad , c=۴ \quad , \frac{\Delta}{۴a}=۶$ ← ۹

$y=a(x-۲)^۲+۶ \longrightarrow ۴=۴a+۶ \rightarrow$

$a=-\frac{۱}{۲} \rightarrow y=-\frac{۱}{۲}(x^۲-۴x+۴)+۶ \rightarrow$

$y=-\frac{x^۲}{۲}+۲x+۴=0 \longrightarrow \frac{\alpha+\beta}{\alpha\beta}=\frac{۴}{-۸}=\boxed{-\frac{۱}{۲}}$

$\leftarrow -۸a - ۸ + ۳a^۲ + ۶a + ۳ = ۰ \rightarrow ۳a^۲ - ۲a - ۱ = ۰$ $\leftarrow ۱۰$

$a = ۱ \,,\, a = -۱/۳ \longrightarrow x^۲ - ۸x + ۱۲ = ۰ \longrightarrow (x-۲)(x-۶) = ۰$

$\longrightarrow x = ۲, ۶$

$\longrightarrow x^۲ - ۸/۳\, x + ۴/۳ = ۰ \rightarrow \Delta > ۰$ ✓

$\longrightarrow$ $x = ۲/۳, ۶$