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تاریخ

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$$f(x) = \begin{cases} x^3 - 8 & ; x > a \\ 12x - 20 & ; x \leq a \end{cases}$$

$$\begin{aligned} x^3 - 8 & \xrightarrow{\text{معادله معکوبی}} \text{یک به یک} \\ 12x - 20 & \xrightarrow{\text{معادله معکوبی}} \text{یک به یک} \end{aligned} \left. \begin{array}{l} x \leq a \\ x > a \end{array} \right\} \text{اشتراک نمی} \quad -1$$

$$12a - 20 = a^3 - 8 \rightarrow a^3 - 12a + 20 - 8 = 0 \rightarrow a^3 - 12a + 12 = 0 \xrightarrow{a=2}$$

$$(2)^3 - 12(2) + 12 = 0 \xrightarrow{\text{تقسیم بر } a-2} a^3 - 12a + 12 = (a-2)(a^2 + 2a - 4) = 0$$

$$(a+2)(a-2) = 0 \quad \begin{cases} \rightarrow a = -2 \\ \rightarrow a = 2 \end{cases}$$

$$f(x) = 3x + k \rightarrow y = 3x + k \rightarrow y = 3x - 10 \quad -2$$

$$f^{-1}(x) \Rightarrow x = 3y + k \Rightarrow f^{-1}(2) \Rightarrow 2 = 3(y) + k \rightarrow k = -10$$

$$f(2) = 3(2) - 10 = 6 - 10 = -4$$

$$f(f(x)) = f(3f(x) - 10) = f(3(3x - 10) - 10) = f(9x - 30 - 10) =$$

$$f(f(x)) = 9x - 40$$

$$A(2a, a) \xrightarrow{\text{تبادل}} A(a, 2a) \quad f(a) = 2a$$

$$2a = \frac{a(a)}{a-1} \rightarrow 2a^2 - 2a = a^2 \rightarrow a^2 - 2a = 0 \rightarrow a(a-2) = 0 \rightarrow$$

$$a = 0, a = 2$$

$$f = \{(3, \omega), (\epsilon, \nu), (\nu, 2), (9, 4)\}$$

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$$g = \{(3, 2), (1, 4), (9, \omega)\}$$

$$f^{-1} = \{(\omega, 3), (\nu, \epsilon), (2, \nu), (4, 9)\}$$

$$g^{-1} = \{(2, 3), (4, 1), (\omega, 9)\}$$

$$\omega) f \circ f^{-1} \rightarrow \omega \rightarrow 3 \rightarrow \omega \mid \nu \rightarrow \epsilon \rightarrow \nu \mid 2 \rightarrow \nu \rightarrow 2 \mid 4 \rightarrow 9 \rightarrow 4$$

$$\{(0, 0), (\nu, \nu), (2, 2), (4, 4)\}$$

$$ب) f^{-1} \circ f \rightarrow 3 \rightarrow \omega \rightarrow 3 \mid \epsilon \rightarrow \nu \rightarrow \epsilon \mid \nu \rightarrow 2 \rightarrow \nu \mid 9 \rightarrow 4 \rightarrow 9$$

$$\{(3, 3), (\epsilon, \epsilon), (\nu, \nu), (9, 9)\}$$

$$ج) f \circ g^{-1} \rightarrow 2 \rightarrow 3 \rightarrow \omega \mid 4 \rightarrow 1 \rightarrow \emptyset \mid \omega \rightarrow 9 \rightarrow 4$$

$$\{(2, \omega), (\omega, 4)\}$$

$$د) g^{-1} \circ f \rightarrow 3 \rightarrow \omega \rightarrow 9 \mid \epsilon \rightarrow \nu \rightarrow \emptyset \mid \nu \rightarrow 2 \rightarrow 3 \mid 9 \rightarrow 4 \rightarrow 1$$

$$\{(3, 9), (\nu, 3), (9, 1)\}$$

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$$f = \{(2, \epsilon), (\epsilon, 1), (4, 0)\}$$

$$g = \{(2, 1), (\epsilon, 3), (4, \omega)\} \rightarrow g^{-1} = \{(1, 2), (3, \epsilon), (\omega, 4)\}$$

$$h = \{(1, 2), (3, \epsilon), (\epsilon, 1), (\omega, 1)\}$$

$$f \circ g^{-1} \rightarrow 1 \rightarrow 2 \rightarrow \epsilon \quad 3 \rightarrow \epsilon \rightarrow 1 \quad \underbrace{\omega \rightarrow 4 \rightarrow \omega}_{\text{خروج، مسیری نه}}$$

$$\{(1, \epsilon), (3, 1), (\omega, 0)\} \rightarrow 1 \notin Df \rightarrow Df \circ g^{-1} = \{3, \omega\}$$

$$D_h = \{1, 3, \epsilon, \omega\} \rightarrow D_h \cap Df \circ g^{-1} = \{3, \omega\}$$

$$\omega \neq \text{خروج} \rightarrow \omega \text{ مسیری نه} \rightarrow h = 3 \checkmark$$

$$\frac{h(3)}{f \circ g^{-1}(3)} = \frac{\epsilon}{1} = \frac{1}{\epsilon} \Rightarrow \{(3, \frac{1}{\epsilon})\}$$

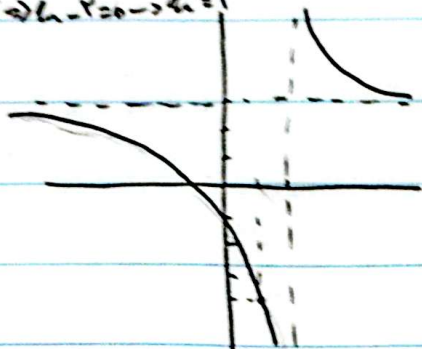
$$y = \frac{3x+1}{x-2} \rightarrow \text{تابع معکوساً متناسب}$$

$$y = \frac{3(1)+1}{1-2} = -4$$

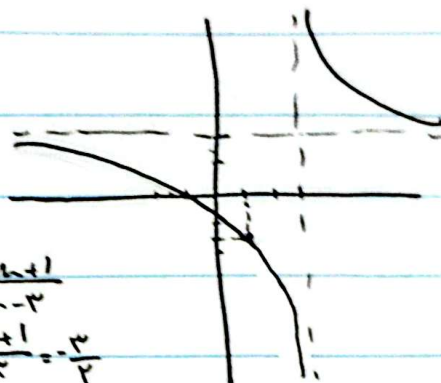
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$$\text{افقی} = \frac{3}{1} = 3$$

$$\text{عمودی} \Rightarrow x-2=0 \Rightarrow x=2$$



معکوس



$$y = \frac{3x+1}{x-2}$$

$$y = \frac{3(1)+1}{1-2} = -4$$

$$y = \frac{3x+1}{x-2} \xrightarrow{\text{معکوس}} x = \frac{3y+1}{y-2} \rightarrow xy - 2x = 3y+1 \rightarrow xy - 3y = 2x+1 \rightarrow y(x-3) = 2x+1$$

$$\rightarrow y = \frac{2x+1}{x-3} = f^{-1}(x) \quad \text{افقی} = \frac{2}{1} = 2 \quad \text{عمودی} \Rightarrow x-3=0 \Rightarrow x=3$$

-v

x	x < 1	1 < x < 3	x > 3
y	$-\frac{x+1}{x-2}$	$\frac{x-1}{x-2}$	$\frac{x-1}{x-2}$

$$f(x) \begin{cases} -2; x < 1 \\ 2x-4; 1 \leq x \leq 3 \\ 2; x > 3 \end{cases}$$

تابع ثابت: $y=2$ یا $y=-2$ است

بزرگترین بازه $\rightarrow$ $[1, 3]$ $\rightarrow$ $f(x) = 2x-4$ $\rightarrow$ $f^{-1}(x) = \frac{x+4}{2}$

$$\text{طول بازه} \Rightarrow b-a = 3-1 = 2$$

$$f^{-1}(x) \Rightarrow f(x) = 2x-4 \quad 1 \leq x \leq 3 \Rightarrow f^{-1}(x) \Rightarrow x = \frac{y+4}{2}$$

$$2y = x+4 \rightarrow y = \frac{x+4}{2}$$

$$\text{بر } f(x) \rightarrow f(1) = 2(1)-4 = -2, \quad f(3) = 2(3)-4 = 2$$

$$\text{پس } f^{-1}(x) = \frac{x+4}{2} \quad D = [-2, 2]$$

پریا حسینی

به ازای کوچکترین $n=1$ زیرا توان فرد است.

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$$f(n) = \begin{cases} n^3 + \varepsilon & ; n \geq 1 \\ \varepsilon n - 1 & ; n \leq 0 \end{cases} \quad R = [\omega, +\infty) \quad R = (-\infty, -1]$$

بیشترین مقدار در $n=0$ می ده

$$n \geq 1 \rightarrow y_1 = y_2 \rightarrow n_1^3 + \varepsilon = n_2^3 + \varepsilon \rightarrow n_1^3 = n_2^3 \rightarrow n_1 = n_2$$

در دامنه ی خود یک به یک است.

$$n \leq 0 \rightarrow y_1 = y_2 \rightarrow \varepsilon n_1 - 1 = \varepsilon n_2 - 1 \rightarrow \varepsilon n_1 = \varepsilon n_2 \rightarrow n_1 = n_2$$

در دامنه ی خود یک به یک است.

$$n \rightarrow R_1 \cap R_2 = [\omega, +\infty) \cap (-\infty, -1] = \emptyset$$

اشتراک بردها

یک به یک است.

$$f^{-1}(n) = \begin{cases} \sqrt[3]{n-\varepsilon} & ; n \geq \omega \\ \frac{n+1}{\varepsilon} & ; n \leq -1 \end{cases}$$

$$y = n^3 + \varepsilon \rightarrow n = \sqrt[3]{y-\varepsilon} \rightarrow y^3 = n - \varepsilon \rightarrow y = \sqrt[3]{n-\varepsilon}$$

$$y = \varepsilon n - 1 \rightarrow n = \varepsilon y - 1 \rightarrow \varepsilon y = n + 1 \rightarrow y = \frac{n+1}{\varepsilon}$$

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$$y = n^3 - \frac{(n+1)^3}{n+3} \xrightarrow{\text{دارون}} n = y^3 - \frac{(y+1)^3}{y+3} \rightarrow$$

$$n = y^3 - \frac{y^3+1+3y^2+3y}{y+3} \rightarrow n = \frac{y^3+3y^2-y^3-1-3y^2-3y}{y+3} \rightarrow n = \frac{-3y-1}{y+3} \rightarrow$$

$$ny + 3n = -3y - 1 \rightarrow ny + 3y = -3n - 1 \rightarrow y(n+3) = -3n - 1 \rightarrow$$

$$y = \frac{-3n-1}{n+3} = \frac{an+b}{cn+d} \quad \frac{-4n-2}{2n+4} \rightarrow \begin{cases} a=-4 \\ b=-2 \end{cases} \quad \begin{cases} c=2 \\ d=4 \end{cases}$$

 $(\times \frac{2}{2})$

$$f^{-1}(b) = \frac{-3(-2)-1}{-2+3} = \omega$$

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برای این

- 1.

$$f(n) = \frac{n}{n^r + 1} \rightarrow y = \frac{n}{n^r + 1} \rightarrow f^{-1}(n) \Rightarrow n = \frac{y}{y^r + 1} \rightarrow$$

$$ny^r + n = y \rightarrow ny^r - y + n = 0$$

$$\Delta = (-1)^r - 4(n)(n) = 1 - 4n^2$$

$$n = \frac{1 \pm \sqrt{1 - 4n^2}}{2n} \quad 1 - 4n^2 \geq 0 \rightarrow 1 \geq 4n^2 \rightarrow \frac{1}{4} \geq n^2 \rightarrow$$

$$-\frac{1}{2} \leq n \leq \frac{1}{2} \rightarrow D f^{-1}(n) \Rightarrow \left[-\frac{1}{2}, \frac{1}{2}\right]$$