

1- $f(x) = \begin{cases} x^2 - 4 & ; x > a \\ 12x - 20 & ; x \leq a \end{cases}$ $a = ?$

$x^2 - 4 \xrightarrow{\text{مشتق}} 2x > 0 \rightarrow \text{متزايد} \rightarrow \text{نريد أن يكون } x > 0 \wedge x \leq -a = \emptyset$
 $12x - 20 \rightarrow + \text{مشتق} \rightarrow \text{متزايد} \rightarrow \text{نريد أن يكون } x > 0 \wedge x \leq -a = \emptyset$

$x = a$ in $x \leq a \rightarrow \max_{x \leq a} (12x - 20) = 12a - 20 \leq a^2 - 4$
 $\min_{x > a} (x^2 - 4) = a^2 - 4$
 $a^2 - 12a + 14 \geq 0 \rightarrow (a-2)^2 (a+4) \geq 0$

$a + 4 \geq 0 \rightarrow a \geq -4$

2- $f(x) = 3x + \frac{k}{x-10}$, $f^{-1}(7) = 4 \rightarrow f(4) = 7 \rightarrow 3(4) + \frac{k}{4-10} = 7 \rightarrow k = -10$

الف) $f(7)$

$\rightarrow 3 \times 7 - 10 = 21 - 10 = 11$

ب) $f(f(x)) \rightarrow (3f(x) - 10) \rightarrow 3(3x - 10) - 10 = 9x - 40 \rightarrow f(f(x)) = 9x - 40$

3- $A(a, a)$

$f(x) = \frac{ax}{x-1}$ $a = ?$

معلوم
 $(a, 2a)$

$f(x) = \frac{ax}{x-1} \rightarrow f(a) = 2a \rightarrow 2a = \frac{a^2}{a-1}$

$a^2 = 2a^2 - 2a \rightarrow a^2 - 2a = 0 \rightarrow a = 0$
 $a = 2$

4- $f = \{(3, 5), (4, 7), (7, 2), (9, 4)\}$ $g = \{(3, 2), (8, 9), (9, 5)\}$

الف) $f \circ f^{-1} \rightarrow \{(5, 5), (4, 4), (2, 2), (7, 7)\}$

ب) $f^{-1} \circ f \rightarrow \{(3, 3), (4, 4), (7, 7), (9, 9)\}$

ج) $f \circ g^{-1} \rightarrow \{(5, 4), (2, 5)\}$

د) $g^{-1} \circ f \rightarrow \{(3, 9), (9, 8), (7, 3)\}$

$$f = \{(3, 4), (4, 8), (6, 0)\} \quad g = \{(3, 1), (4, 3), (6, 4)\} \quad h = \{(1, 2), (3, 4), (4, 1), (5, 0)\}^{-1}$$

$$\frac{h}{f \circ g^{-1}} \rightarrow D_h = \{1, 3, 4, 5\} \rightarrow \text{است} \rightarrow (3)$$

$$f \circ g^{-1} \rightarrow \{(1, 4), (3, 8), (4, 0)\} \rightarrow D_{f \circ g^{-1}} = \{3, 4\}$$

$$\frac{h(3)}{f \circ g^{-1}(3)} = \frac{4}{8} = \frac{1}{2} \rightarrow (3, \frac{1}{2})$$

$$y = \frac{3x+1}{x-2}$$

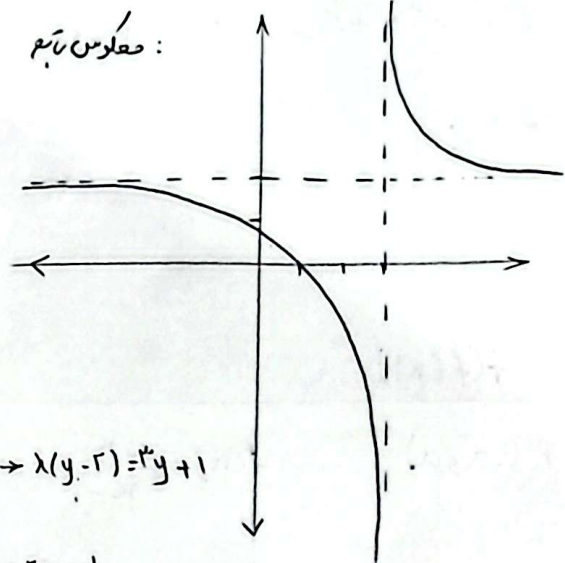
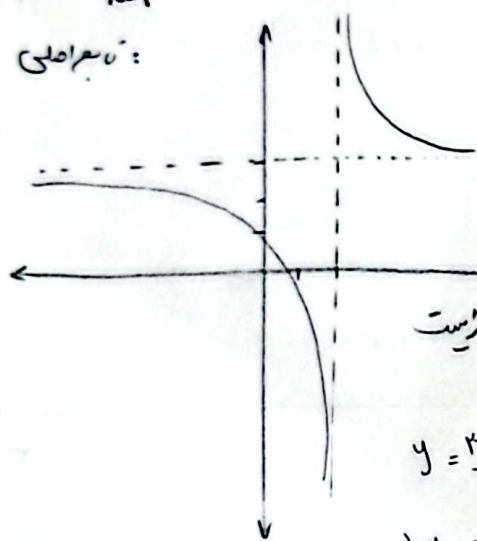
نیم اصلی:

مقلوب = 3

ضابطه مقلوب = 3

معوار مقلوب = 3

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نیم برعکس است
است پس مقلوب برعکس است

$$y = \frac{3x+1}{x-2} \xrightarrow{\text{مقلوب}} x = \frac{3y+1}{y-2} \rightarrow x(y-2) = 3y+1$$

$$xy - 2x = 3y + 1 \rightarrow xy - 3y = 2x + 1$$

$$y(x-3) = 2x + 1 \rightarrow y = \frac{2x+1}{x-3} = f^{-1}(x)$$

ضابطه نیم مقلوب

$$f(x) = |x-1| - |x-3| \rightarrow [a, b] \rightarrow \text{نیم برعکس} \quad \text{؟ ضابطه مقلوب}$$

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$$x=1 \quad x=3$$

$$|x-1| - |x-3|$$

$$\begin{array}{c|c|c} 1 & 3 & \\ \hline -x+1 & x-1 & x-3 \\ \hline -x+1 & x-1 & x-3 \\ \hline -2 & 2x-4 & 2 \\ \hline \text{صفری} & \text{صفری} & \text{صفری} \end{array}$$

مقدارهای 1 و 3
نیم برعکس است
max = 3
min = 1
[1, 3]

$$b-a = 3-1 = 2$$

$$f(x) = 2x - 4 \xrightarrow{\text{مقلوب}} x = 2y - 4 \rightarrow y = \frac{x+4}{2} = f^{-1}(x) \rightarrow D \in [-2, 2]$$

$$f(n) = \begin{cases} n^2 + \varepsilon & ; n \geq 1 \\ \varepsilon n - 1 & ; n \leq 0 \end{cases}$$

$$f^{-1}(n) = \begin{cases} \sqrt{n - \varepsilon} & ; n \geq \varepsilon \\ \frac{n+1}{\varepsilon} & ; n \leq -1 \end{cases}$$

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$$\begin{aligned} n \geq 1 &\rightarrow y_1 = y_2 \rightarrow n_1^2 + \varepsilon = n_2^2 + \varepsilon \rightarrow n_1^2 = n_2^2 \rightarrow n_1 = n_2 \rightarrow R = [\varepsilon, +\infty) \\ n \leq 0 &\rightarrow y_1 = y_2 \rightarrow \varepsilon n_1 - 1 = \varepsilon n_2 - 1 \rightarrow n_1 = n_2 \rightarrow R = (-\infty, -1] \end{aligned}$$

این دو بازه را می‌توان به هم پیوست و به صورت $R = (-\infty, -1] \cup [\varepsilon, +\infty)$ نوشت.

$$n^2 + \varepsilon \xrightarrow{\text{مقلوب}} = \sqrt{n - \varepsilon}$$

$$\varepsilon n - 1 \xrightarrow{\text{مقلوب}} = \frac{n+1}{\varepsilon}$$

$$f^{-1}(n) = \begin{cases} \sqrt{n - \varepsilon} & ; n \geq \varepsilon \\ \frac{n+1}{\varepsilon} & ; n \leq -1 \end{cases}$$

$$f(n) = n^2 - \frac{(n+1)^2}{n+2}$$

$$\xrightarrow{\text{مقلوب}} f^{-1}(n) = \frac{an+b}{cn+d}$$

$$f^{-1}(b) \quad b < 0$$

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$$n^2 - \frac{(n+1)^2}{n+2} \xrightarrow{\text{مقلوب}} n - y^2 - \frac{(y+1)^2}{y+2} \rightarrow \frac{y^2 + y^2 - y^2 - 1 - y^2 - 2y}{y+2} = \frac{-y-1}{y+2}$$

$$n(y+2) = -y-1 \rightarrow y = \frac{-2n-1}{n+2} \rightarrow \frac{-4n-2}{2n+4} = \frac{an+b}{cn+d} \rightarrow b = -2$$

$$f^{-1}(b) = \frac{-4(-2)-2}{2(-2)+4} \Rightarrow 5$$

$$f(n) = \frac{n}{n^2+1}$$

$$f^{-1}(n) = ?$$

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$$y = \frac{n}{n^2+1} \rightarrow n = \frac{y}{y^2+1} \rightarrow n(y^2+1) = y \rightarrow ny^2 + n = y \rightarrow y = \frac{1 \pm \sqrt{1-4n^2}}{2n}$$

$$\frac{n}{n^2+1} \rightarrow \text{فقط برای } n > 0 \text{ و } n < 0 \text{ به ازای هر دو مقدار درست نیست}$$

$$f^{-1}(n) = \frac{1 \pm \sqrt{1-4n^2}}{2n}$$

$$Df^{-1}(n) = -\frac{1}{n} \leq n \leq \frac{1}{n} \leftarrow D \geq |n| \leq \frac{1}{n}$$