

-1

$$\text{if } f(x) = \begin{cases} x^2 - \varepsilon & ; x > a \\ 12x - 20 & ; x \leq a \end{cases} \rightarrow \text{نیمه باز} \quad a = ?$$

(2)

$$x^2 - \varepsilon \xrightarrow{\text{مثبت}} x^2 > 0 \rightarrow \text{مثبت} \rightarrow \text{نیمه باز} \Rightarrow x > 0 \wedge x \leq -a = \emptyset$$

$$12x - 20 \rightarrow + \text{مثبت} \rightarrow \text{مثبت} \rightarrow \text{نیمه باز}$$

$$x = a \text{ in } x \leq a \rightarrow \max_{x \leq a} (12x - 20) = 12a - 20 \quad \left. \begin{array}{l} 12a - 20 \leq a^2 - \varepsilon \\ \min_{x > a} (x^2 - \varepsilon) = a^2 - \varepsilon \end{array} \right\} a^2 - 12a + 20 \geq 0 \rightarrow (a-2)^2 (a+4)$$

$$\text{نیمه باز} \rightarrow a + \varepsilon \geq 0 \quad \text{چون } (a-2)^2 \geq 0 \Rightarrow a \geq -\varepsilon$$

-2

$$f(x) = 3x + \frac{k}{-10}, \quad f^{-1}(7) = \varepsilon \rightarrow f^{-1}(x) = 3y + k \rightarrow 7 = 3\varepsilon + k \rightarrow k = -10$$

(2)

الف) $f(7)$

$$\rightarrow 3 \times 7 - 10 = 21 - 10 = 11$$

$$\text{ب) } f(f(x)) \rightarrow (3f(x) - 10) \rightarrow 3(3x - 10) - 10 = 9x - 40 \rightarrow f(f(x)) = 9x - 40$$

-3

$$A(a, a) \quad f(x) = \frac{ax}{x-1} \quad a = ?$$

مقوس
(a, 2a)

$$f(x) = \frac{ax}{x-1} \rightarrow f(a) = 2a \rightarrow 2a = \frac{a^2}{a-1}$$

(1, 1.5)

$$a^2 = 2a^2 - 2a \rightarrow a^2 - 2a = 0 \rightarrow a = 0 \quad \times$$

$$\rightarrow a = 2 \quad \checkmark$$

-4

$$f = \{(3, 5), (4, 7), (7, 2), (9, 4)\} \quad g = \{(3, 2), (8, 4), (9, 5)\}$$

(2)

$$\text{الف) } f \circ f^{-1} \rightarrow \{(5, 5), (4, 4), (2, 2), (7, 7)\}$$

$$\text{ب) } f^{-1} \circ f \rightarrow \{(3, 3), (4, 4), (7, 7), (9, 9)\}$$

$$\text{ج) } f \circ g^{-1} \rightarrow \{(5, 4), (2, 5)\}$$

$$\text{د) } g^{-1} \circ f \rightarrow \{(3, 9), (9, 8), (7, 3)\}$$

$$f = \{(3, 4), (4, 8), (6, 0)\} \quad g = \{(3, 1), (4, 3), (6, 4)\} \quad h = \{(1, 2), (3, 4), (4, 1), (5, 0)\}^{-1}$$

$$\frac{h}{f \circ g^{-1}} \rightarrow D_h = \{1, 3, 4, 5\} \rightarrow \text{انتهی} \rightarrow (3)$$

(۱)

$$f \circ g^{-1} \rightarrow \{(1, 4), (3, 8), (4, 0)\} \rightarrow D_{f \circ g^{-1}} = \{3, 4, 1\}$$

$$\frac{h(3)}{f \circ g^{-1}(3)} = \frac{4}{8} = \frac{1}{2} \rightarrow \left\{ \left(3, \frac{1}{2}\right), \left(1, \frac{1}{2}\right), \left(5, \frac{1}{2}\right) \right\}$$

$$y = \frac{3n+1}{n-2}$$

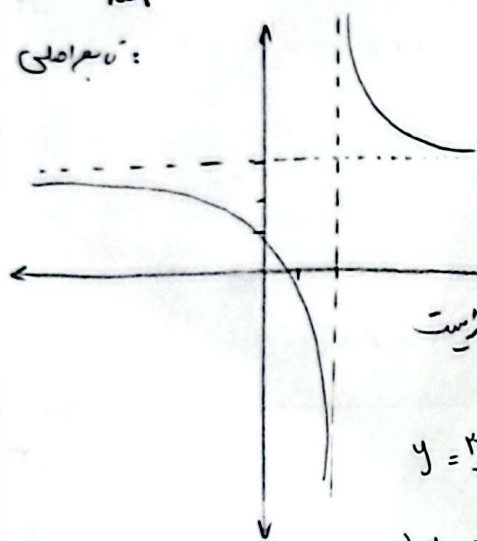
نیم اصلی:

مقلوب = 3

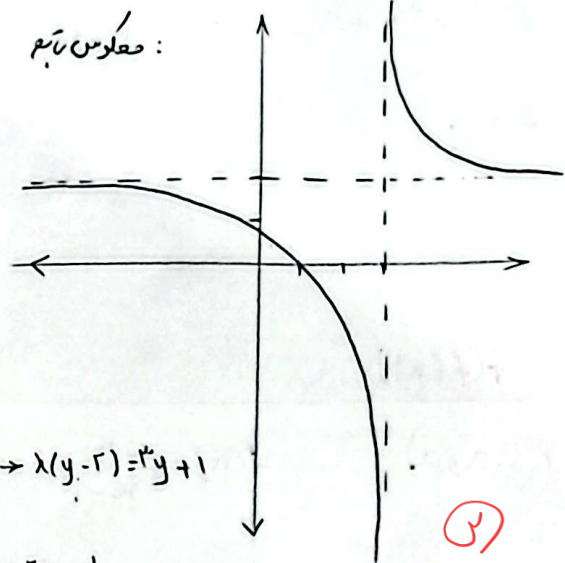
ضابطه مقلوب = 3

معوار مقلوب = 3

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نیم برعکس است
است پس مقلوب برعکس است



(۲)

$$y = \frac{3n+1}{n-2} \xrightarrow{\text{مقلوب}} n = \frac{3y+1}{y-2} \rightarrow \lambda(y-2) = 3y+1$$

$$\lambda y - 2\lambda = 3y + 1 \rightarrow \lambda y - 3y = 2\lambda + 1$$

$$y(\lambda - 3) = 2\lambda + 1 \rightarrow y = \frac{2\lambda + 1}{\lambda - 3} = f^{-1}(\lambda)$$

ضابطه نیم مقلوب

$$f(n) = |n-1| - |n-3| \rightarrow [a, b] \rightarrow \text{نیم برعکس} \quad \text{؟ ضابطه مقلوب}$$

-7 (۳)

$$n=1 \quad n=3$$

$$|n-1| - |n-3| \rightarrow$$

$$\begin{array}{ccc} 1 & 3 & \\ \hline -n+1 & -n+3 & \\ \hline -1 & 2 & \\ \hline \end{array}$$

صفری صفری صفری

مقدارهای مثبت و ۱ و ۳
نیم برعکس است
max = [1, 3]

$$b-a = 3-1 = 2$$

$$f(n) = 2n-4 \xrightarrow{\text{مقلوب}} n = 2y-4 \rightarrow y = \frac{n+4}{2} = f^{-1}(n) \rightarrow D \in [-2, 2]$$

$$f(n) = \begin{cases} n^2 + \varepsilon & n \geq 1 \\ \varepsilon n - 1 & n \leq 0 \end{cases}$$

$$f^{-1}(n) = \begin{cases} \sqrt{n-\varepsilon} & n \geq \varepsilon \\ \frac{n+1}{\varepsilon} & n \leq -1 \end{cases}$$

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$$\begin{aligned} n \geq 1 &\rightarrow y_1 = y_2 \rightarrow n_1^2 + \varepsilon = n_2^2 + \varepsilon \rightarrow n_1^2 = n_2^2 \rightarrow n_1 = n_2 \rightarrow R = [\varepsilon, +\infty) \\ n \leq 0 &\rightarrow y_1 = y_2 \rightarrow \varepsilon n_1 - 1 = \varepsilon n_2 - 1 \rightarrow n_1 = n_2 \rightarrow R = (-\infty, -1] \end{aligned}$$

این دو بازه را می‌توانیم به هم وصل کنیم و به یک بازه تبدیل می‌دهیم.

$$n^2 + \varepsilon \xrightarrow{\text{مقلوب}} = \sqrt{n-\varepsilon}$$

$$\varepsilon n - 1 \xrightarrow{\text{مقلوب}} = \frac{n+1}{\varepsilon}$$

$$f^{-1}(n) = \begin{cases} \sqrt{n-\varepsilon} & n \geq \varepsilon \\ \frac{n+1}{\varepsilon} & n \leq -1 \end{cases}$$

(۵)

$$f(n) = n^2 - \frac{(n+1)^2}{n+2}$$

$$\xrightarrow{\text{مقلوب}} f^{-1}(n) = \frac{an+b}{cn+d}$$

$$f^{-1}(b)$$

$$b < 0$$

-۹

$$n^2 - \frac{(n+1)^2}{n+2} \xrightarrow{\text{مقلوب}} n - y^2 - \frac{(y+1)^2}{y+2} \rightarrow \frac{y^2 + y^2 - y^2 - 1 - y^2 - 2y}{y+2} = \frac{-y-1}{y+2}$$

$$n(y+2) = -y-1 \rightarrow y = \frac{-2n-1}{n+2} \rightarrow \frac{-4n-2}{2n+4} = \frac{an+b}{cn+d} \rightarrow b = -2$$

(۵)

$$f^{-1}(b) = \frac{-4(-2)-2}{2(-2)+4} \Rightarrow (5)$$

$$f(n) = \frac{n}{n^2+1}$$

$$f^{-1}(n) = ?$$

(۵)

-۱۰

$$y = \frac{n}{n^2+1} \rightarrow n = \frac{y}{y^2+1} \rightarrow n(y^2+1) = y \rightarrow ny^2 + n = y \rightarrow y = \frac{1 \pm \sqrt{1-4n^2}}{2n}$$

$$\frac{n}{n^2+1} \rightarrow \text{مقلوب برای } n > 0 \text{ و } n < 0 \text{ به ازای هر دو مقدار یکسان نیست}$$

$$f^{-1}(n) = \frac{1 \pm \sqrt{1-4n^2}}{2n}$$

$$Df^{-1}(n) = -\frac{1}{n} \leq n \leq \frac{1}{n} \leftarrow D \geq |n| \leq \frac{1}{n}$$