آقای کرمانی — پاسخنامه تست ریاضی شماره ۱۲ — یازدهم ریاضی A

۱)

$f(x)=\begin{cases} x^3-4 & ; x\geq a \\ 12x-20 & ; x<a \end{cases}$

$x=a \begin{cases} a^3-4 \\ 12a-20 \end{cases} \Rightarrow a^3-4 \geq 12a-20 \Rightarrow a^3-12a+16\geq 0 \Rightarrow (a-2)(a^2+2a-8)\geq 0$

$(a-2)(a+4)(a-2)\geq 0 \quad \begin{array}{c|c|c} -4 & & 2 \\ - & + & + \end{array} \Rightarrow a \in (-4,2)\cup(2,+\infty)$

۲)

$f^{-1}(2)=4 \; , \; f(x)=3x+k \Rightarrow f^{-1}(2)=4 \Rightarrow f(4)=2 \Rightarrow 12+k \Rightarrow k=-10$

الف) $f(7)=3x-10 \Rightarrow 21-10=11$

ب) $f(f(x))=3(3x-10)-10 \Rightarrow 9x-40$

۳)

$A=(2a,a) \; , \; f(x)=\frac{ax}{x-1} \Rightarrow f^{-1}(x) \Rightarrow x=\frac{ay}{y-1} \Rightarrow xy-x=ay$

$-x=ay-xy \Rightarrow -x=y(a-x) \Rightarrow y=\frac{-x}{a-x} \Rightarrow y=\frac{x}{x-a} \xrightarrow{(2a,a)} a=\frac{2a}{2a-a} \Rightarrow a=2$

۴)

$g=\{(3,2),(8,6),(9,5)\} \; , \; f=\{(3,5),(4,7),(7,2),(9,9)\}$

الف) $f\circ f^{-1}=\{(5,5),(7,7),(2,2),(9,9)\}$

ب) $f^{-1}\circ f=\{(3,3),(4,4),(7,7),(9,9)\}$

ج) $f\circ g^{-1}=\{(2,5),(5,9)\}$

د) $g^{-1}\circ f=\{(3,9),(7,3),(9,8)\}$

۵)

$f=\{(2,4),(4,8),(6,0)\} \; , \; g=\{(2,1),(4,2),(6,5)\} \; , \; h=\{(1,2),(3,4),(4,1),(5,1)\}$

$\frac{h}{f\circ g^{-1}}=\{(5,\cancel{\tfrac{1}{0}}),(3,\cancel{\tfrac{4}{8}}),(1,\cancel{\tfrac{2}{4}})\}$ نیست $\Rightarrow \frac{h}{f\circ g^{-1}}=\{(3,\frac{1}{2}),(1,\frac{1}{2})\}$

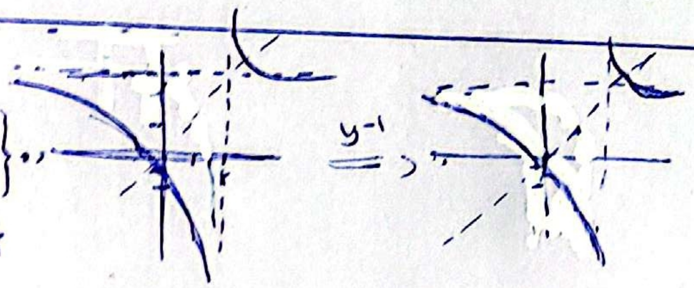
۶)

$y=\frac{3x+1}{x-2} \Rightarrow$

$\frac{3x}{x}=3$ افقی

$x-2=0 \Rightarrow x=2$ عمودی

$x=0 \Rightarrow -\frac{1}{2}$



$x=\frac{3y+1}{y-2} \Rightarrow xy-2x=3y+1 \Rightarrow -2x-1=3y-xy \Rightarrow$

$\frac{-2x-1}{3-x}=y \Rightarrow y=\frac{2x+1}{x-3} \xrightarrow{x=0} \frac{-1}{3}$

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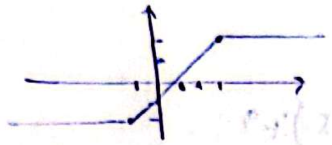
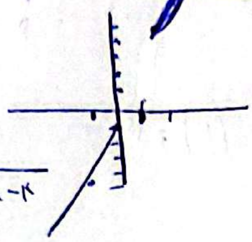
$f(x)=|x-1|-|x-3| \Rightarrow f(x)\begin{cases} 2 & x\geq 3 \\ 2x-4 & 1\leq x<3 \Rightarrow x=2y-4 \Rightarrow x+4=2y \Rightarrow y=\frac{x}{2}+2 \\ -2 & x<1 \end{cases}$



8

$f(x)=\begin{cases} x^3+4 \; ; x\geq 1 \\ 4x-1 \; ; x\leq 0 \end{cases}$



$f^{-1}(x)=\begin{cases} x=y^3+4 \Rightarrow x-4=y^3 \Rightarrow y=\sqrt[3]{x-4} \\ x=4y-1 \Rightarrow x+1=4y \Rightarrow \frac{x+1}{4}=y \end{cases}$

$f^{-1}(x)=\begin{cases} y=\sqrt[3]{x-4} \; ; x\geq 5 \\ \frac{x+1}{4} \; ; x\leq -1 \end{cases}$

$D_f=(-\infty,\cancel{0}]\cup[1,+\infty)$

$R_f=(-\infty,-1]\cup[5,+\infty) \Rightarrow$

$D_{f^{-1}}=(-\infty,-1]\cup[5,+\infty)=$

$R_{f^{-1}}=(-\infty,0]\cup[1,+\infty)$

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$f(x)=x^2-\frac{(x+1)^3}{x+3} \quad , f^{-1}(x)=\frac{ax+b}{2x+d} \quad f^{-1}(5)=?$

$f(x)=x^2-\frac{x^3+3x^2-x^3-1-3x^2-3x}{x+3}=\frac{-3x-1}{x+3}$

$f^{-1}(x)=\cancel{y} = \frac{-3y-1}{y+3} \Rightarrow xy+3x=-3y-1 \Rightarrow 3x+1=-3y-xy \Rightarrow \frac{3x+1}{-x-3}=y\times\left(\frac{-2}{-2}\right) \Rightarrow$

$y=\frac{-6x-2}{2x+6} \Rightarrow b=-2 \Rightarrow f^{-1}(-2)=\frac{12-2}{2}=5$

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$f(x)=\frac{x}{x^2+1} \Rightarrow f^{-1}(x)=x=\frac{y}{y^2+1} \Rightarrow xy^2+x=y \Rightarrow xy^2-y+x=0$

$y=\frac{1\pm\sqrt{1-4x^2}}{2x}$