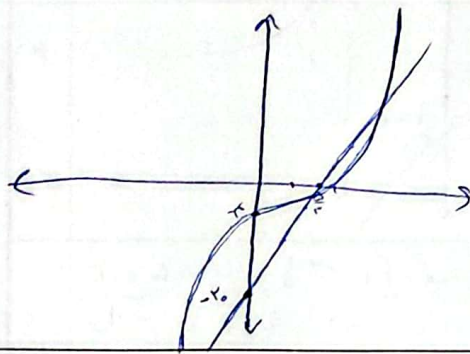




$$f(x) = \begin{cases} x^2 - 4, & x > a \\ 12x - 10, & x \leq a \end{cases}$$

$$a^2 - 4 > 12a - 10$$

$$(a-2)^2 (a+4) > 0$$

$$a+4 > 0 \rightarrow \boxed{a > -4}$$

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$$y = 2x + k \rightarrow 12 + k = 2 \rightarrow k = -10 \rightarrow 2x - 10 = y \rightarrow 2y - 10 = x$$

$$\boxed{y = \frac{x+10}{2}} \leftarrow \text{دارد}$$

الف) $f(v) = 2v - 10 = 11$

ب) $f(f(x)) = f(2x - 10) = 2(2x - 10) - 10 = \boxed{4x - 30}$

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$$\frac{ax}{x-1} = y \Rightarrow ax = yx - y \Rightarrow y = x(y-a) \Rightarrow x = \frac{y}{y-a} \Rightarrow y = \frac{x}{x-a} \Rightarrow$$

$$\frac{ya}{a} = a \Rightarrow \boxed{a=2}$$

تابع $f(x)$ بازای a می ده پس درون
آن بازای a به a می دهد.

$$\frac{ya}{a} = a \Rightarrow ya = a^2 \Rightarrow a^2 - 2a = 0 \Rightarrow a(a-2) = 0 \Rightarrow a = \boxed{2}$$

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$$f^{-1}(x) = \{(5, 2), (7, 4), (2, 7), (4, 9)\} \quad g^{-1}(x) = \{(2, 5), (4, 8), (5, 9)\}$$

الف) $f \circ f^{-1} = \{(5, 5), (7, 7), (2, 2), (4, 4)\}$

ب) $f^{-1} \circ f = \{(2, 2), (7, 7), (4, 4), (9, 9)\}$

ج) $f \circ g^{-1} = \{(2, 5), (5, 6)\}$

د) $g^{-1} \circ f = \{(5, 9), (7, 8), (9, 11)\}$

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$$g^{-1} = \{(1, 2), (3, 4), (5, 6)\} \Rightarrow h = \{(1, 2), (3, 4), (4, 1), (5, 1)\}$$

$$f \circ g^{-1} = \{(1, 4), (3, 8), (5, 10)\} \Rightarrow \frac{h}{f \circ g^{-1}} = \frac{(1, 2), (3, 4), (5, 1)}{(1, 4), (3, 8), (5, 10)} \Rightarrow$$

$$\frac{h}{f \circ g^{-1}} = \frac{2}{4}, \frac{4}{8}, \frac{1}{10} \Rightarrow \frac{h}{f \circ g^{-1}} = \boxed{\frac{1}{2}}$$

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$y = \frac{x+1}{x-2}$ 	$y = \frac{x+1}{x-2} \Rightarrow yx - 2y = x+1 \Rightarrow x = \frac{-y-1}{y-1} \Rightarrow y = \frac{x+1}{x-2}$ 	6
$\begin{cases} x > 2 & x-1-x+2 = -1 \\ 1 \leq x \leq 2 & x-1+x-2 = 2x-3 \\ x < 1 & -x+1+x-2 = -1 \end{cases}$ 	تابع ما در بازه $[1, 2]$ یک به یک هست پس: $b=1, a=1$ $2x-3=y \Rightarrow 2x=y+3 \Rightarrow x = \frac{y+3}{2} \Rightarrow y = \frac{x+3}{2}$ 	7
	$x^2-1=y \Rightarrow x^2=y+1 \Rightarrow x = \sqrt{y+1} \Rightarrow y = x^2-1$ $x = \sqrt{y+1} \Rightarrow y = x^2-1$ 	8
$\frac{x^2(x+1) - x^2 - 1 - \frac{x^2-1}{x+1}}{x+1} = y$ $\frac{x^2 + \frac{x^2-1}{x+1} - x^2 - 1 - \frac{x^2-1}{x+1}}{x+1} \Rightarrow \frac{-x-1}{x+1} = y \Rightarrow y = \frac{-x-1}{x+1}$ 	$yx + cy = -cx - 1 \Rightarrow yx + cx = -1 - cy \Rightarrow x = \frac{-1-cy}{y+c}$ $\frac{-cx-1}{x+c} = \frac{ax+b}{cx+d}$ $\begin{cases} a=-1 \\ b=-1 \\ c=1 \\ d=1 \end{cases} \Rightarrow \frac{-x-1}{x+1} = \frac{-x-1}{x+1}$ 	9
$f(x) = \frac{x}{x^2+1} \Rightarrow y(x^2+1) = x \Rightarrow yx^2 - x + y = 0 \Rightarrow \Delta = 1-4y^2$ $x = \frac{1 \pm \sqrt{1-4y^2}}{2y} \Rightarrow 1-4y^2 \geq 0 \Rightarrow 4y^2 \leq 1 \Rightarrow -\frac{1}{2} \leq y \leq \frac{1}{2}$ $f(x)^{-1} = \frac{1 \pm \sqrt{1-4y^2}}{2y} \Rightarrow f(x)^{-1} = \frac{1 + \sqrt{1-4y^2}}{2y} \Rightarrow x \in [-\frac{1}{2}, \frac{1}{2}], x \neq 0$		10