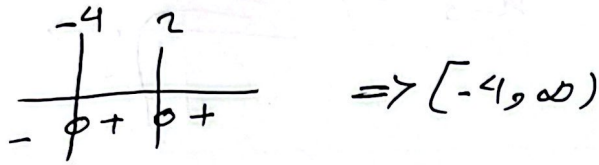


$$12a - 20 \leq a^3 - 4 \rightarrow a^3 - 12a + 16 \geq 0 \rightarrow (a-2)^2(a+4) \geq 0$$



۱

$$f^{-1}(u) \Rightarrow u = 3y + k \Rightarrow 3y = -k + u \Rightarrow y = \frac{u - k}{3} \xrightarrow{u=2} \frac{2 - k}{3} = 4$$

$$\Rightarrow k = -10$$

۲

الف) $f(7) = 21 - 10 = 11$

ب) $f \circ f(x) = 9x - 30 - 10 = 9x - 40$

$$f^{-1}(u) \Rightarrow xg = u = ay \Rightarrow y = \frac{u}{u-a}$$

$$\Rightarrow \left| \frac{2a}{a} \right| \Rightarrow a = \frac{2a}{2a-a} \Rightarrow a^2 - 2a = 0$$

$$\Rightarrow a = 0 \text{ و } a = 2$$

۳

الف) $\{(5,5), (7,6), (2,2), (6,6)\}$

ب) $\{(3,3), (4,4), (7,7), (9,9)\}$

ج) $\{(2,5), (5,6)\}$

د) $\{(3,9), (7,3), (9,8)\}$

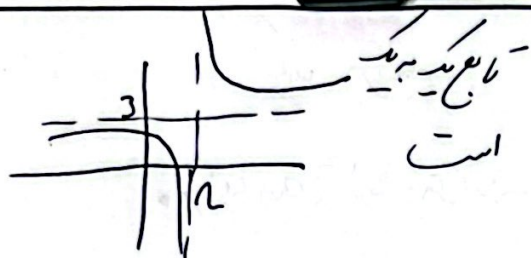
۴

$$f \circ g^{-1} = \{(1,4), (3,8), (5,0)\}$$

$$h = \{(1,2), (3,4), (4,1), (5,1)\}$$

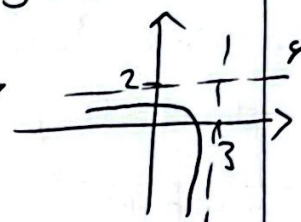
$$\frac{h}{f \circ g^{-1}} = \{(1,2/4), (3,4/8)\}$$

۵



$$f^{-1}(x) \Rightarrow x = \frac{3y+1}{y-2}$$

$$\Rightarrow y = \frac{2x+1}{x-3}$$



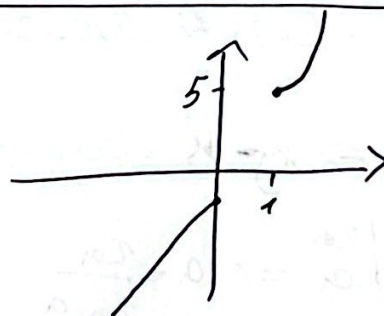
$$[a, b] = [1, 3]$$

$$\begin{cases} 3a+b=2 \\ a+b=-2 \end{cases} \Rightarrow a=2, b=-4$$

$$y=2x-4 \Rightarrow x=2y-4 \rightarrow y=\frac{x+4}{2}$$

$$f^{-1}(x) = \begin{cases} \frac{x+1}{4} & x \leq -1 \\ \sqrt[3]{x-4} & x > 5 \end{cases}$$

$$R_f(x) = D_{f^{-1}(x)}$$



$$f(x) = \frac{3x+1}{-3x} \xrightarrow{f^{-1}(x)} x = \frac{3y+1}{-3+y} \Rightarrow y = \frac{3x+1}{-x-3}$$

$$\Rightarrow \frac{6x-2}{2x+6} = \frac{ax+b}{cx+d} \Rightarrow a=-6, b=-2, c=6$$

$$f^{-1}(2) = \frac{2-12}{10} = -1$$

$$x^2 + x - y = 0$$

$$y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$$

$$1-4x^2 \geq 0 \Rightarrow x^2 \leq \frac{1}{4}$$

$$\Rightarrow [-\frac{1}{2}, \frac{1}{2}]$$

$$f^{-1}(x) = \begin{cases} \frac{1 + \sqrt{1-4x^2}}{2x} & -\frac{1}{2} \leq x < 0 \\ \frac{1 - \sqrt{1-4x^2}}{2x} & 0 < x \leq \frac{1}{2} \end{cases}$$