

(1) $f(n) = \begin{cases} n^3 - \varepsilon & ; n > a \\ 12n - 20 & ; n \leq a \end{cases} \Rightarrow$ $a^3 - \varepsilon > 12a - 20$ $\Rightarrow a^3 - 12a + 14 > 0 \Rightarrow (a-2)^2(a+7) > 0$ $\Rightarrow a > -\varepsilon$

(2) $f(n) = 3n + k \Rightarrow y = 3n + k \Rightarrow n = \frac{y-k}{3} \Rightarrow y = \frac{n-k}{3}$
 $f^{-1}(y) = \varepsilon \Rightarrow \frac{y-k}{3} = \varepsilon \Rightarrow y-k = 12 \Rightarrow k = -10$

الف) $f(v) = 3(v) - 10 = 11$

ب) $f(f(n)) = f(3n + k) = 3(3n + k) + k = 9n + 3k + k = 9n - 10$

(3) $A(ka, a)$

$f(n) = \frac{an}{n-1} \Rightarrow y = \frac{an}{n-1} \Rightarrow n = \frac{ay}{y-1} \Rightarrow ny - n = ay \Rightarrow ny - ay = n$
 $y(n-a) = n \Rightarrow y = \frac{n}{n-a} \Rightarrow a = \frac{2a}{2a-a} \Rightarrow a = 2$

(4) $f = \{(3, \omega), (4, \nu), (7, 2), (9, 4)\}$ $g = \{(3, 2), (1, 4), (9, \omega)\}$

$f^{-1} = \{(\omega, 3), (\nu, 4), (2, 7), (4, 9)\}$ $g^{-1} = \{(2, 3), (4, 1), (\omega, 9)\}$

الف) $f \circ f^{-1} = \{(\omega, \omega), (\nu, \nu), (2, 2), (4, 4)\}$ $g^{-1} \circ f = \{(3, 9), (\nu, 3), (9, 1)\}$

ب) $f^{-1} \circ f = \{(3, 3), (4, 4), (\nu, \nu), (9, 9)\}$

ج) $f \circ g^{-1} = \{(2, \omega), (\omega, 4)\}$

(5) $f = \{(2, 4), (4, 8), (6, 0)\}$ $g = \{(2, 1), (4, 3), (6, \omega)\}$

$h = \{(1, 2), (3, 4), (4, 1), (\omega, 1)\}$ $g^{-1} = \{(1, 2), (3, 4), (\omega, 6)\}$

$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{4}), (3, \frac{1}{4})\}$

تعریف $\frac{1}{0} = 0$

$f \circ g = \{(1, 4), (3, 8), (\omega, 0)\}$

(6)

$y = \frac{r_{n+1}}{n-r}$ $\Rightarrow$ $y = \frac{r_{n+1}}{n-r}$ $\Rightarrow$ $ny - rn = r_{n+1}$

$\Rightarrow ny - ry = r_{n+1} \Rightarrow y(n-r) = r_{n+1} \Rightarrow y = \frac{r_{n+1}}{n-r}$

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(7)

$f(n) = |n-1| - |n-3| \Rightarrow [a, b]$

$[a, b] = [1, 3]$

$\Rightarrow$ $y = r_{n-1}$

$\Rightarrow$ $n = ry - 1 \Rightarrow y = \frac{n+1}{r}$

(8)

$f(n) = \begin{cases} n^3 + 1 & ; n \geq 1 \\ \varepsilon n - 1 & ; n \leq 0 \end{cases}$

$n \geq 1 \Rightarrow y = n^3 + 1$

$\Rightarrow n = \sqrt[3]{y-1} = y$

$n \leq 0 \Rightarrow y = \varepsilon n - 1$

$\Rightarrow n = \frac{y+1}{\varepsilon} = y$

$f^{-1}(n) = \begin{cases} \sqrt[3]{n-1} & ; n \geq 1 \\ \frac{n+1}{\varepsilon} & ; n \leq 0 \end{cases}$

(9)

$f(n) = n^2 - \frac{(n+1)^3}{n+3} \Rightarrow y = n^2 - \frac{(n+1)^3}{n+3}$

$f^{-1}(n) = \frac{an+b}{rn+d}$

$\Rightarrow n = \frac{y^2 - (y+1)^3}{y+3} \Rightarrow n = \frac{y^2 - y^3 - 3y^2 - 3y - 1}{y+3}$

$\Rightarrow n = \frac{-(1+3y)}{3+y} \Rightarrow 3n + 2y = -1 - 3y \Rightarrow ny + 3y = -3n - 1$

$y(n+3) = -3n - 1 \Rightarrow y = \frac{-3n-1}{n+3}$

$\Rightarrow \frac{an+b}{rn+d} = \frac{-3n-1}{n+3}$

$\Rightarrow \frac{an+b}{rn+d} = \frac{-3n-1}{n+3}$

(10)

$f(n) = \frac{n}{nr+1} \Rightarrow y = \frac{n}{nr+1} \Rightarrow n = \frac{y}{yr-1}$

$\Rightarrow ny^r - n = y \Rightarrow ny^r - y = n \Rightarrow ny^r - y - n = 0$

$\Delta = 1 - 4(n)(-n) = 1 + 4n^2$

$y = \frac{1 \pm \sqrt{1+4n^2}}{2n}$