

(1) $f(n) = \begin{cases} n^3 - \varepsilon & ; n > a \\ 12n - 20 & ; n \leq a \end{cases} \Rightarrow$ $a^3 - \varepsilon > 12a - 20$ $\Rightarrow a^3 - 12a + 14 \geq 0 \Rightarrow (a-2)^2(a+7) \geq 0$ $\Rightarrow a \geq -2$

فرض $n = a$ $\Rightarrow a^3 - \varepsilon > 12a - 20$ $\Rightarrow a^3 - 12a + 14 \geq 0$ $\Rightarrow (a-2)^2(a+7) \geq 0$ $\Rightarrow a \geq -2$

(2) $f(n) = 3n + k \Rightarrow y = 3n + k \Rightarrow n = \frac{y-k}{3} \Rightarrow y = \frac{n-k}{3}$

$f^{-1}(y) = \varepsilon \Rightarrow \frac{y-k}{3} = \varepsilon \Rightarrow y-k = 12 \Rightarrow k = -10$

الف) $f(v) = 3(v) - 10 = 11$

ب) $f(f(n)) = f(3n + k) = 3(3n + k) + k = 9n + 3k + k = 9n - 10$

$A(1a, a)$

$f(n) = \frac{an}{n-1} \Rightarrow y = \frac{an}{n-1} \Rightarrow n = \frac{ay}{y-1} \Rightarrow ny - n = ay \Rightarrow ny - ay = n$

$y(n-a) = n \Rightarrow y = \frac{n}{n-a} \Rightarrow a = \frac{2a}{2a-a} \Rightarrow a = 2$

$f = \{(3, \infty), (4, 7), (7, 2), (9, 4)\}$ $g = \{(3, 2), (1, 4), (9, \infty)\}$

$f^{-1} = \{(\infty, 3), (7, 4), (2, 7), (4, 9)\}$ $g^{-1} = \{(2, 3), (4, 1), (\infty, 9)\}$

الف) $f \circ f^{-1} = \{(\infty, \infty), (7, 7), (2, 2), (4, 4)\}$ $g^{-1} \circ f = \{(3, 9), (7, 3), (9, 1)\}$

ب) $f^{-1} \circ f = \{(3, 3), (4, 4), (7, 7), (9, 9)\}$

ج) $f \circ g^{-1} = \{(2, \infty), (\infty, 4)\}$

$f = \{(2, 4), (4, 8), (6, 0)\}$ $g = \{(2, 1), (4, 3), (6, \infty)\}$

$h = \{(1, 2), (3, 4), (4, 1), (\infty, 1)\}$ $g^{-1} = \{(1, 2), (3, 4), (\infty, 4)\}$

$\frac{h}{f \circ g^{-1}} = \{(1, \frac{1}{4}), (3, \frac{1}{4})\}$

$f \circ g^{-1} = \{(1, 4), (3, 8), (\infty, 0)\}$

(6)

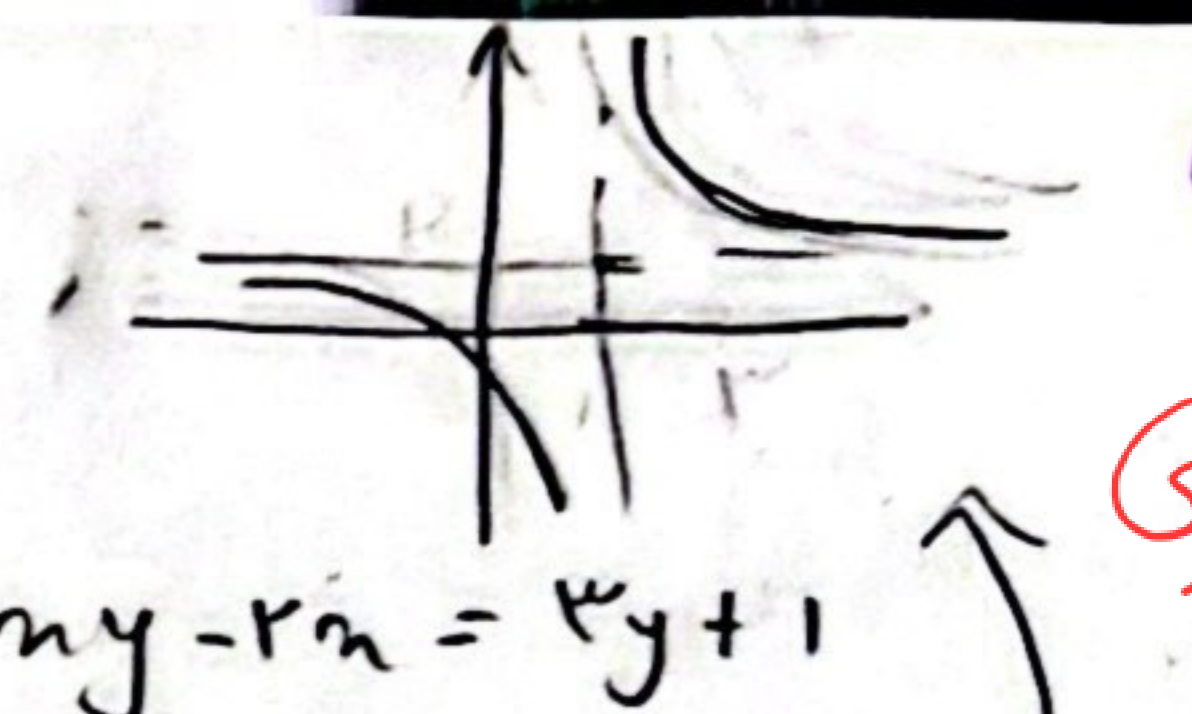
$y = \frac{r_{n+1}}{n-r}$ → معکوس

$n = \frac{ry+1}{y-r} \Rightarrow ny - rn = ry+1$

$\Rightarrow ny - ry = rn+1 \Rightarrow y(n-r) = rn+1 \Rightarrow y = \frac{rn+1}{n-r}$

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معکوس

(7)

$f(n) = |n-1| - |n-r| \Rightarrow [a, b]$

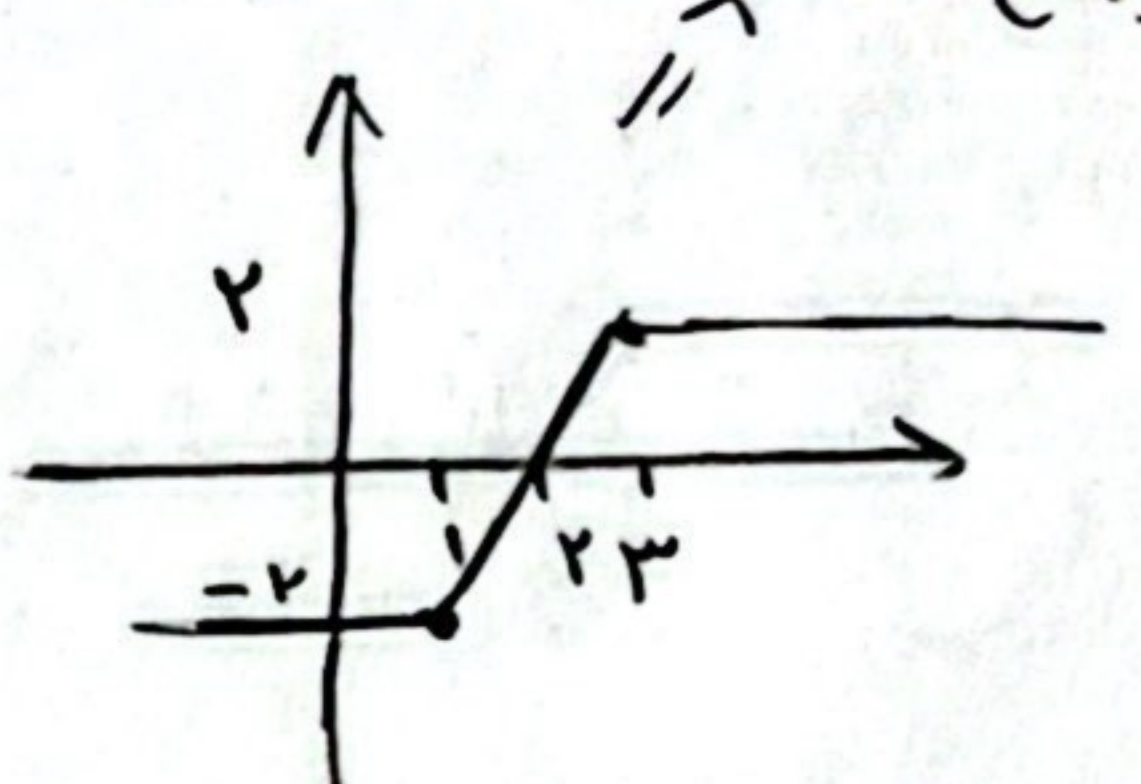
$[a, b] = [1, r]$

$\Rightarrow$

$y = rn - \varepsilon$

$n = \frac{y+\varepsilon}{r} \Rightarrow y = \frac{rn+\varepsilon}{r}$

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(8)

$f(n) = \begin{cases} n^r + \varepsilon & ; n \geq 1 \\ \varepsilon n - 1 & ; n \leq 0 \end{cases}$

$n \geq 1 \Rightarrow y = n^r + \varepsilon$

معکوس $\hookrightarrow n = y^{\frac{1}{r}} + \varepsilon \Rightarrow \sqrt[r]{n-\varepsilon} = y$

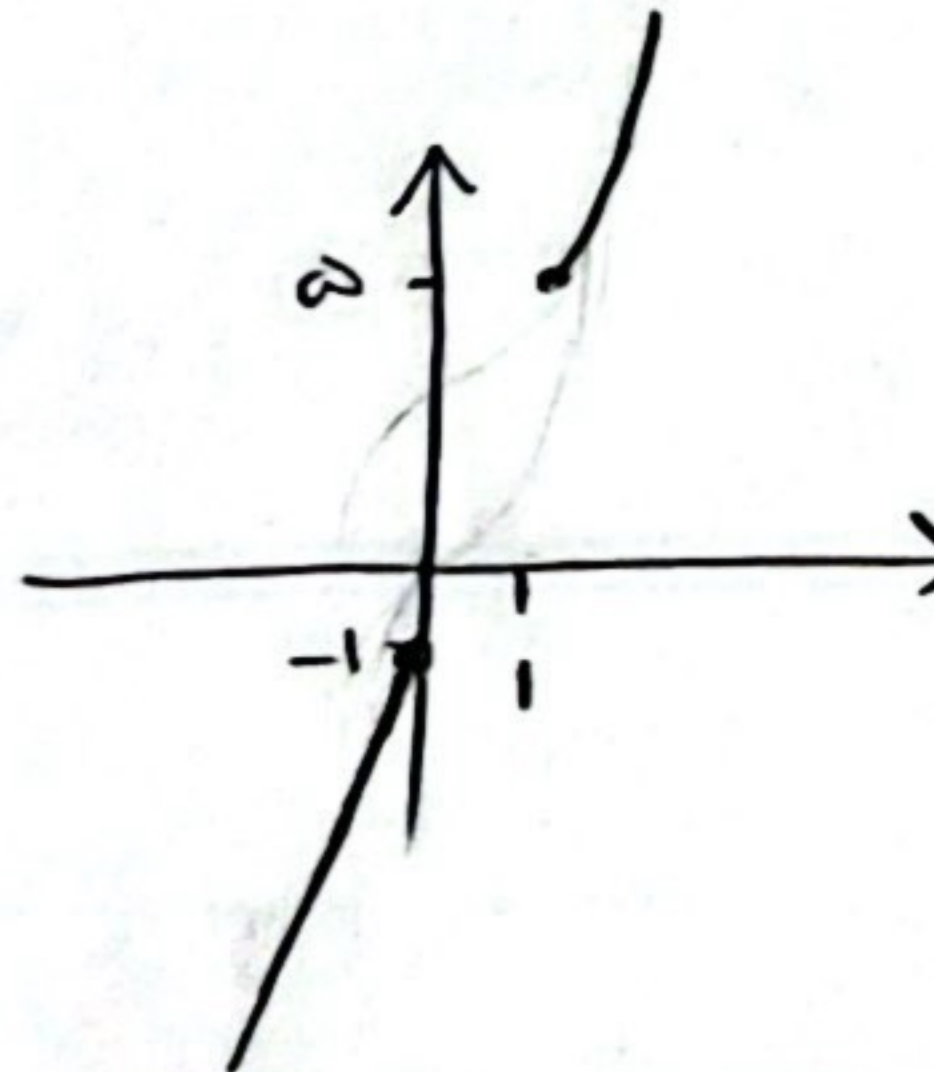
$n \leq 0 \Rightarrow y = \varepsilon n - 1$

معکوس $\hookrightarrow n = \frac{y+1}{\varepsilon} \Rightarrow \frac{n+1}{\varepsilon} = y$

$f^{-1}(n) = \begin{cases} \sqrt[r]{n-\varepsilon} & ; n \geq \varepsilon \\ \frac{n+1}{\varepsilon} & ; n \leq -1 \end{cases}$

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واردن و نیندر و بایر



(9)

$f(n) = n^r - \frac{(n+1)^r}{n+r} \Rightarrow y = n^r - \frac{(n+1)^r}{n+r}$

$f^{-1}(b) = \frac{an+b}{rn+d}$

$f^{-1}(b)$

$b < 0$

$n = y^r - \frac{(y+1)^r}{y+r} \Rightarrow n = \frac{y^r + ry - y^r - 1 - ry - r^2}{y+r}$

$\Rightarrow n = \frac{-(1+ry)}{r+y} \Rightarrow rn + ry = -1 - ry \Rightarrow ny + ry = -rn - 1$

$y(n+r) = -rn - 1 \Rightarrow y = \frac{-rn-1}{n+r}$

$d = r, b = -1 \Rightarrow \frac{an+b}{rn+d} = \frac{-rn-1}{n+r}$

$\Rightarrow \frac{an+b}{rn+d} = \frac{-rn-1}{n+r}$

(10)

$f(n) = \frac{n}{nr+1} \Rightarrow y = \frac{n}{nr+1}$

معکوس $\Rightarrow n = \frac{y}{y^r-1}$

$\Rightarrow ny^r - n = y \Rightarrow ny^r - y = n \Rightarrow ny^r - y - n = 0$

$\Delta = 1 - 4(n)(-n) = 1 + 4n^2$

$y = \frac{1 \pm \sqrt{1+4n^2}}{2n}$