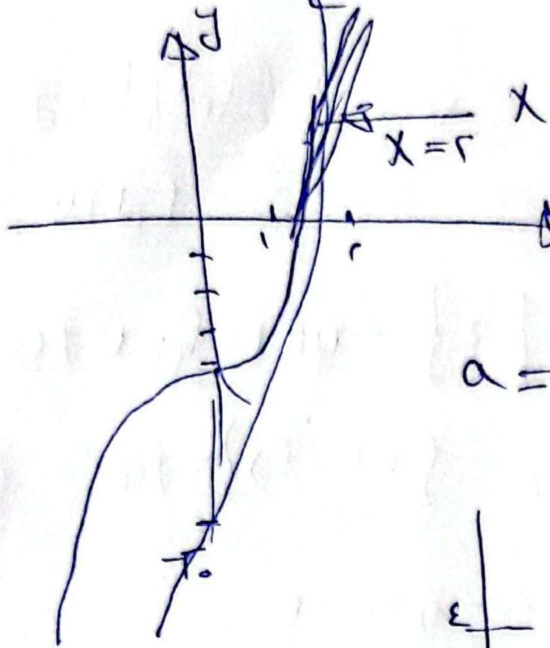
نگار اصفهانی

۱۶,۵

① $f(x)=\begin{cases} x^2-4 & x\geq a \\ 12x-20 & x\leq a\end{cases}$

۵



$x^2-4x=12x-20$

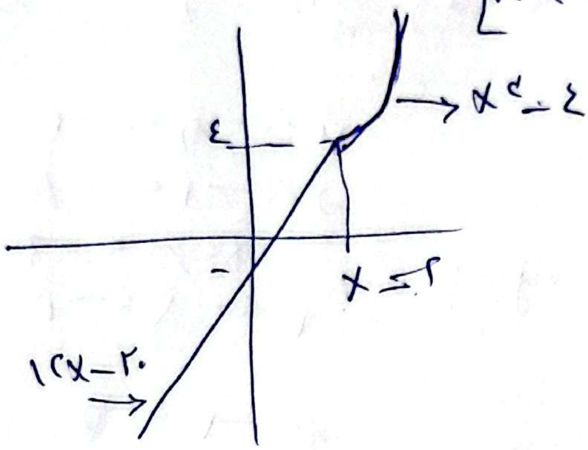
$\to x^2-12x+16=0$

$x=2$

$a^3-4\geq 12a-20$

$\to a\in[-4,+\infty)$

$a=2 \to \begin{cases} x\geq 2 & x^2-4 \\ x\leq 2 & 12x-20\end{cases}$



② $f(2)=4 \to f(4)=2 \to f(4)=2\times 4+k=2$ ②

$k=-10$

۲

$f(x)=3x-10$

الف $\to f(7)=21-10=11 \to f(7)=11$

ب $f(x)=3x-10$

$f(f(x))=3(3x-10)-10=9x-40 \to f(f(x))=9x-40$

⑤ $A(2a,a) \to A'(a,2a)$

f^{-1} روی نمودار f روی $\to 2a=\dfrac{a^2}{a-1}$ ③ ۲

$2a^2-2a=a^2$

$a^2-2a=0$ $\begin{cases} a=0 & \times \\ a=2 & \checkmark\end{cases}$

(ع)

(أ) $f \circ f^{-1}(x)$

$\{(٥,٥)(٧,٧)(٢,٢)(٦,٦)\}$ ١

$٢ \to ٥ \to \times$
$٤ \to ٧ \to ٢ \to f\circ f^{-1}(x) = \{(٤,٢)\}$
$٧ \to ٢ \to \times$
$٩ \to ٦ — \times$

ب $f^{-1} \circ f(x)$

$٢ \to ٥ \to ٢$
$٤ \to ٧ \to ٤$
$٧ \to ٢ \to ٧$
$٩ \to ٦ \to ٦$

~~$f \circ f(x) = \{(٢,٢)(٤,٤)(٧,٧)(٦,٦)\}$~~

$\{(٣,٣)(٤,٤)(٧,٧)(٩,٩)\}$

(2.) $f \circ g^{-1}(x)$

g^{-1}
$٢ \to ٢ \to ٥$
$٦ \to ٨ \to \times$
$٥ \to ٩ \to ٦$

$\to f\circ g^{-1} = \{(٢,٥)(٥,٦)\}$

(د) $g^{-1} \circ f$

$٢ \to ٥ \to ٩$
$٤ \to ٧ \to \times$
$٧ \to ٢ \to ٢$
$٩ \to ٦ \to ٨$

$g^{-1}\circ f = \{(٢,٩)(٧,٢)(٩,٨)\}$

(ه) $Df = \{٢,٤,٦\}$ $Dg^{-1} = \{٢,٤,٥\}$ $Dh = \{١,٢,٤,٥\}$

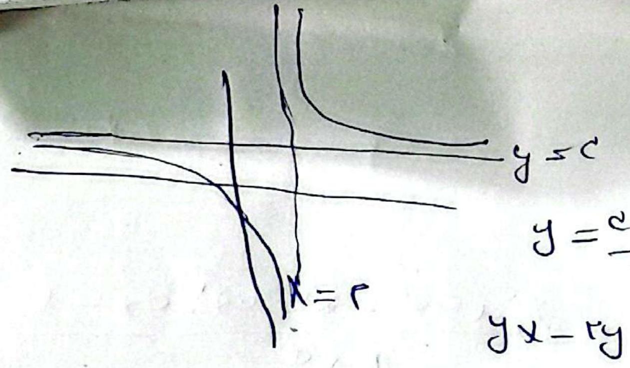
$Df \cap Dg^{-1} \cap Dh = \{٤\}$ $Dg^{-1}\{٢,١,٥\}$

١,٥

$$\frac{h(٤)}{f(g^{-1}(٤))} = \frac{٤}{٨} = \frac{١}{٢}$$

$\{(١,\frac{١}{٢})(٣,\frac{١}{٢})\cancel{(٥,\frac{١}{٠})}\}$

٦



٢

$$y=\frac{٢x+١}{x-٣}$$

$$yx-٣y=٢x+١ \rightarrow yx-٢x=١+٣y$$

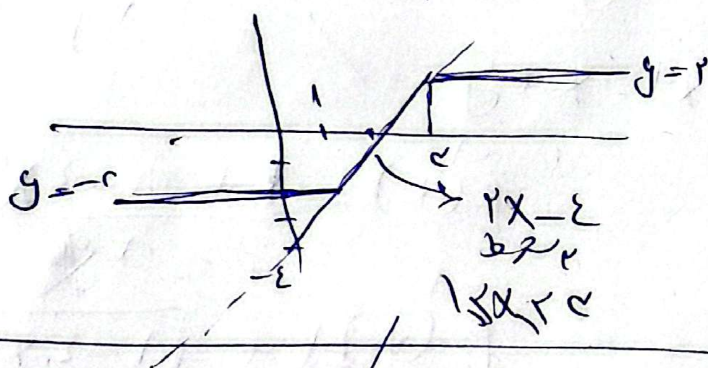
$$x(y-٢)=١+٣y$$

$$f^{-1}(x)=\frac{١+٣x}{x-٢} \longleftarrow x=\frac{١+٣y}{y-٢}$$

٧

١/٥

	١	٢	
$-x+١$ $\ominus$ $-x+٢$	$x-١$ $\ominus$ $-x+٢$	$x-١$ $-$ $x-٢$
-٢	$٢x-٤$	٢



$\rightarrow ٢x-٤$

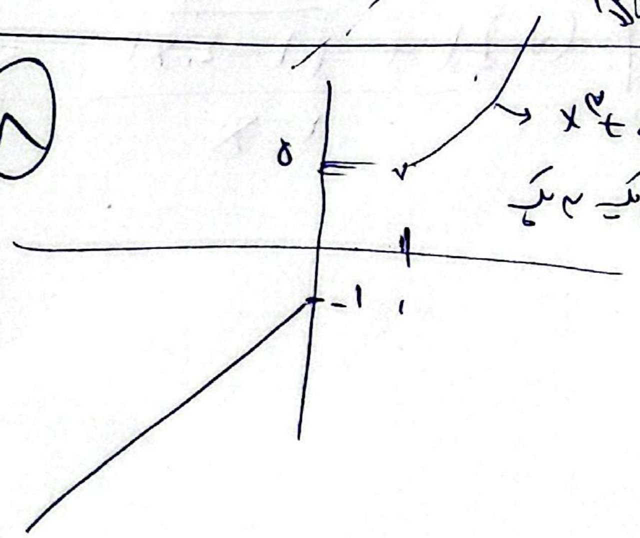
$-٢\le y\le ٢$

$١\le x<٢$

$$y=٢x-٣ \rightarrow x=\frac{y+٣}{٢}$$

$$f^{-1}(x)=\frac{x+٣}{٢}$$

٨



$\rightarrow x^٣+٤$

تابع یک به یک

$$f^{-1}(x)=\begin{cases}\sqrt[٣]{x-٤} & x\geq ٥\\ \frac{x+١}{٤} & x\leq -١\end{cases}$$

٢

۴

$$y=\frac{x^{۲}(x+۳)-(x+1)^{۳}}{(x+۳)}=\frac{\cancel{x^{۳}}+\cancel{۳x^{۲}}-\cancel{x^{۳}}-\cancel{۳x^{۲}}-۳x-1}{x+۳}$$

$$f(x)=\frac{-۳x-1}{x+۳}$$

۲

$$y=\frac{-۳x-1}{x+۳}\rightarrow yx+۳y=-۳x-1$$

$$x(y+۳)=-۳y-1$$

$$f^{-1}(x)=\frac{-۳x-1}{x+۳}=\frac{-۶x-۲}{۲x+۶}\rightarrow \begin{matrix} d=۶\\ b=-۲\\ a=-۶\end{matrix}$$

$$f^{-1}(x)=\frac{-۶x-۲}{۲x+۶},\ f(-۲)=\frac{-۶x-۲-۲}{۲x-۲+۶}=\frac{۱۰}{۲}=۵$$

$$\boxed{f^{-1}(b)=۵}$$

۱۰

$$f(x)=\frac{x}{x^{۲}+1}\rightarrow yx^{۲}+y=x$$

$$yx^{۲}-x+y=0 \quad / \quad x=\frac{+1\pm\sqrt{1-۴y^{۲}}}{۴y}$$

$D_f \rightarrow R$, $R_f: -\frac{1}{۲}\leq y\leq \frac{1}{۲}$ ۲

$$f^{-1}(x)=\frac{\cancel{1}+\sqrt{1-۴\cancel{y}^{۲}}}{۲\cancel{y}}$$

$$f^{-1}(x)=\frac{\pm 1+\sqrt{1-۴x^{۲}}}{۲x}$$