

$$f(x) = \begin{cases} x^3 - 4 & ; x > a \\ 12x - 2 & ; x \leq a \end{cases}$$

$$a^3 - 4 > 12a - 2 \Rightarrow (a-1)^2(a+2) \geq 0 \Rightarrow \boxed{a \geq -2}$$

$$f(x) = 12x + k \quad , \quad f(1) = 1 \Rightarrow f(2) = 2 \Rightarrow f(2) = 12 \cdot 2 + k = 2 \Rightarrow 24 + k = 2 \Rightarrow k = -22$$

$$\text{الف) } f(v) = 12v + k \Rightarrow 12(v) + 1 = 11 \Rightarrow v = 1$$

$$\text{ب) } f(f(m)) \Rightarrow 12(12m + k) + k = 144m + 12k + k = 144m + 13k = 144m - 22$$

$$f(m) = \frac{am}{m-1} \Rightarrow f^{-1}\left(\frac{am}{m-1}\right) = m \quad A(10, a)$$

$$\Rightarrow \frac{am}{m-1} = 10 \Rightarrow \frac{a}{a-1} = 10 \Rightarrow a = 10a - 10 \Rightarrow a - 10a = -10 \Rightarrow -9a = -10 \Rightarrow a = \frac{10}{9}$$

$$f = \{(1, 2), (2, 1), (3, 4), (4, 3)\} \quad g = \{(1, 3), (2, 4), (3, 2), (4, 1)\}$$

$$\text{الف) } f \circ f^{-1} \Rightarrow f^{-1} = \{(2, 1), (1, 2), (4, 3), (3, 4)\} \Rightarrow f \circ f^{-1} = \begin{cases} 2 \rightarrow 1 \rightarrow 2 \\ 1 \rightarrow 2 \rightarrow 1 \\ 4 \rightarrow 3 \rightarrow 4 \\ 3 \rightarrow 4 \rightarrow 3 \end{cases} \Rightarrow \{(2, 2), (1, 1), (4, 4), (3, 3)\}$$

$$\text{ب) } f \circ f \Rightarrow f = \{(2, 1), (1, 2), (4, 3), (3, 4)\} \Rightarrow f \circ f = \begin{cases} 1 \rightarrow 2 \rightarrow 1 \\ 2 \rightarrow 1 \rightarrow 2 \\ 3 \rightarrow 4 \rightarrow 3 \\ 4 \rightarrow 3 \rightarrow 4 \end{cases} \Rightarrow \{(1, 1), (2, 2), (3, 3), (4, 4)\}$$

$$\text{ج) } f \circ g^{-1} \Rightarrow g^{-1} = \{(3, 1), (4, 2), (2, 3), (1, 4)\} \Rightarrow f \circ g^{-1} = \{(3, 2), (4, 1), (2, 4), (1, 3)\}$$

$$\text{د) } g \circ f = \{(1, 3), (2, 4), (3, 2), (4, 1)\} \Rightarrow g \circ f = \{(1, 3), (2, 4), (3, 2), (4, 1)\}$$

$$h = \{(1, 2), (2, 1), (3, 4), (4, 3)\} \quad g = \{(1, 3), (2, 4), (3, 2), (4, 1)\} \quad f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$$

$$\frac{h}{f \circ g^{-1}} = \frac{h}{f} \Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right) \Rightarrow \left(\frac{1}{2}, \frac{1}{2}\right)$$

$$f \circ g^{-1} \Rightarrow g^{-1} = \{(1, 3), (2, 4), (3, 2), (4, 1)\} \Rightarrow f \circ g^{-1} = \{(1, 3), (2, 4), (3, 2), (4, 1)\}$$

$$D_h \cap D_{f \circ g^{-1}} = \{(1, 2), (2, 1)\}$$

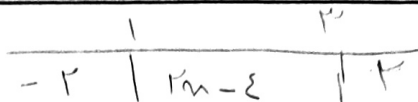
$$y = \frac{r^m + 1}{x - r} \xrightarrow{\text{crossing}} x = \frac{ry + 1}{y - r} \Rightarrow ry - rx = ry + 1 \Rightarrow ry - ry = r + 1$$

$$\Rightarrow y(r - r) = r + 1 \Rightarrow y = \frac{r + 1}{r - r}$$



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$$f(m) = |m - 1| - |m - r|$$

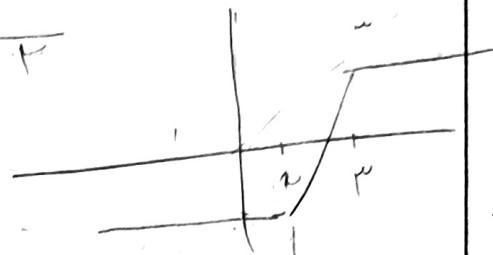


$$m - 1 - m + r = r - m + 1$$

$$r - m - \varepsilon$$

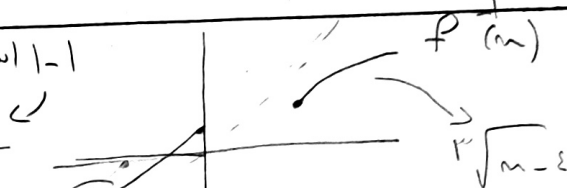
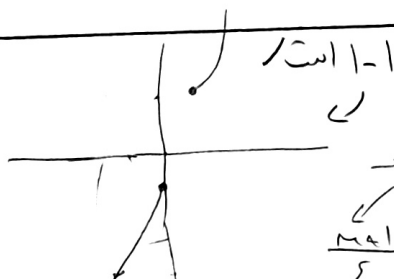
[1, r]

$$\xrightarrow{\text{crossing}} ry - \varepsilon = m \Rightarrow y = \frac{m + \varepsilon}{r} \Rightarrow [-r, 1]$$



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$$f(m) = \begin{cases} m^r + \varepsilon & m \geq 1 \\ \varepsilon m - 1 & m \leq 0 \end{cases}$$



$$m^r + \varepsilon = y \Rightarrow y^r + \varepsilon = m \rightarrow y^r = m - \varepsilon \rightarrow y = \sqrt[r]{m - \varepsilon}$$

$$r - 1 = y \Rightarrow m = ry - 1 \rightarrow \frac{m + 1}{r} = y$$

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$$f(m) = m^r - \frac{(m+1)^r}{m+r} \Rightarrow \frac{m^r + r m^{r-1} - (m^r + r m^{r-1} + r m^{r-2} + \dots + r m + 1)}{m+r} = \frac{-r m + 1}{m+r}$$

$$f^{-1}(m) = \frac{am+b}{cm+d} = \frac{1(rm-1)}{-m+r} \Rightarrow \frac{am+b}{cm+d} = \frac{-rm-1}{rm+r}$$

$$f^{-1}(b) = \frac{-9(-r) - 1}{r(-r) + 4} = \frac{1}{2} = r$$

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$$f(m) = \frac{x}{m^r + 1} \xrightarrow{f(m)} x = \frac{y}{y^r + 1} \Rightarrow ry^r + m = y \Rightarrow ry^r - y = -m$$

$$-ry^r + y = m$$

$$ry^r - y + m = 1 - \varepsilon(r)(m) = 1 - \varepsilon m^r$$

$$f^{-1}(m) = \frac{1 \pm \sqrt{1 - \varepsilon m^r}}{r m} \rightarrow D_{f^{-1}} = \frac{1}{r} \geq |m| \quad m \in \left[-\frac{1}{r}, \frac{1}{r}\right]$$

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