نام و نام خانوادگی: بهار شهسواریان — تکلیف ۱۱ — گروه یازدهم A

۱)

$$f(x)=\begin{cases} x^3-4 & , x>a \rightarrow a^3-4 \\ 12x-20 & , x\le a \rightarrow 12a-20 \end{cases}$$

$f(x) \rightarrow R = (-\infty , 12a-20]$

$\Rightarrow a^3-4 \geq 12a-20 \rightarrow a^3-12a+16 \geq 0$

$\rightarrow (a-2)(a^2+2a-8) \geq 0 \rightarrow (a-2)(a+4)(a-2) \geq 0$

$-4 \quad 2$: $- \mid + \mid +$ $\rightarrow$ $a \geq -4$

۲)

$f(x)=3x+k \quad , \quad f^{-1}(2)=4$

$f^{-1}(x): x=3y+k \rightarrow 3y=x-k \rightarrow y=\frac{x-k}{3}=4$ ، $k=-10$

الف) $f(7)$

$f(7)=21-10=11$

ب) $f(f(x))$

$f(f(x))=9x-40$

$3(3x-10)-10=9x-30-10$

$=9x-40$

۳)

$f(x)=\frac{ax}{x-1} \rightarrow f^{-1}(x): x=\frac{ya}{y-1} \rightarrow y=\frac{-x}{a-x} \quad , \quad y=\frac{x}{x-a}$

$A(2a,a) \rightarrow \frac{2a}{2a-a}=a \rightarrow a=2$

۴)

الف) $f o f^{-1}=\{(5,5)(7,7)(2,2)(6,6)\}$

$5\rightarrow 3 \rightarrow 5$

$7\rightarrow 4 \rightarrow 7$

$2\rightarrow 7 \rightarrow 2$

$6\rightarrow 9 \rightarrow 6$

$\{(5,5)(2,2)(7,7)(6,6)\}$

ب) $f^{-1} o f$

$3\rightarrow 5 \rightarrow 3$

$4\rightarrow 7 \rightarrow 4$

$7\rightarrow 2 \rightarrow 7$

$9\rightarrow 6 \rightarrow 9$

$\{(3,3)(4,4)(7,7)(9,9)\}$

پ) $f o g^{-1}$

$2\rightarrow 3 \rightarrow 5$

$6\rightarrow 8 \rightarrow \times$

$5\rightarrow 9 \rightarrow 6$

$\{(2,5)(5,6)\}$

د) $g^{-1} o f$

$3\rightarrow 5 \rightarrow 9$

$4\rightarrow 7 \rightarrow \times$

$7\rightarrow 2 \rightarrow 3$

$9\rightarrow 6 \rightarrow 8$

$\{(3,9)(7,3)(9,8)\}$

۵)

$f=\{(2,4)(4,8)(6,0)\}$

$g=\{(2,1)(4,3)(6,5)\}$

$h=\{(1,2)(3,4)(4,1)(5,5)\}$

$f o g^{-1}: \{(1,4)(3,8)(5,0)\}$

$f(g^{-1}(x))$:

$1\rightarrow 2 \rightarrow 4$

$3\rightarrow 4 \rightarrow 8$

$5\rightarrow 6 \rightarrow 0$

$\frac{h}{f o g^{-1}}=\{(1,\frac{1}{2})(3,\frac{1}{2})(5, \text{تعریف نشده})\} \rightarrow \{(1,\frac{1}{2})(3,\frac{1}{2})\}$

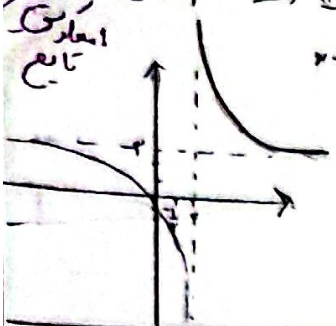
۶

$y=\dfrac{3x+1}{x-2} \rightarrow \dfrac{a}{c}=3$

$x-2 \rightarrow$ ② ریشه‌ی مخرج

$x=0 , y=-\dfrac{1}{2}$

نمودار معکوس تابع



ضابطه‌ی تابع معکوس

$\dfrac{3y+1}{y-2}=x$

$y=-\dfrac{-2x-1}{x-3}$

$y=\dfrac{2x+1}{x-3}$

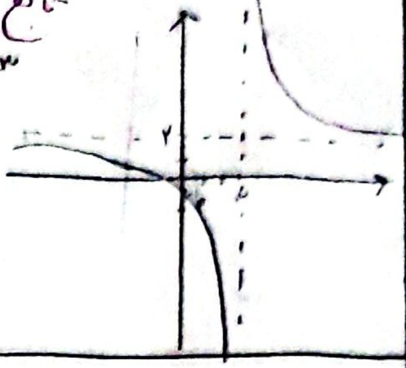
نمودار معکوس تابع

$x=3$ ریشه‌ی مخرج

$\dfrac{a}{c}=2$

$x=0$

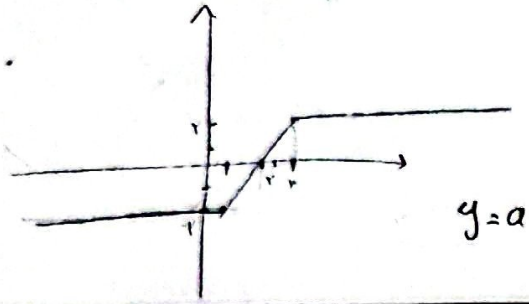
$y=-\dfrac{1}{3}$



۷

$f(x)=|x-1|-|x-3|$

D_f ... ②



$y=2x-4 \xrightarrow{\text{معکوس}} x=2y-4 \rightarrow 2y=x+4$

$y=\dfrac{x+4}{2}$

$[-2,2]$

$\Rightarrow f^{-1}(x)=\dfrac{x+4}{2} , x\in[-2,2]$

$y=ax+b$

$3a+b=2$

$-\quad a+b=-2$

$2a=4 \rightarrow a=2 , b=-4 \; \Big\} \; y=2x-4$

۸

$f(x)=\begin{cases} x^3+4 \\ 4x-1 \end{cases}$

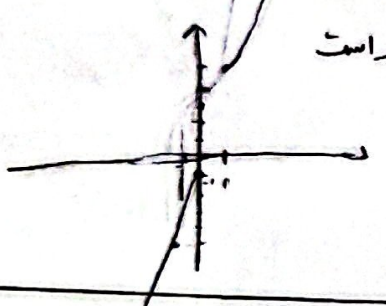
$x\geq 1 \rightarrow R=[5,\infty)$

$x<0 \rightarrow (-\infty,1]$

تابع یک به یک است

$x=y^3+4 \rightarrow y^3=x-4 \rightarrow y=\sqrt[3]{x-4}$

$x=4y-1 \rightarrow 4y=x+1 \rightarrow y=\dfrac{x+1}{4}$



$f^{-1}(x)=\begin{cases} \sqrt[3]{x-4} & x\geq 5 \\ \dfrac{x+1}{4} & x\leq -1 \end{cases}$

۹

$f(x)=x^2-\dfrac{(x+1)^3}{x+3} \rightarrow f(x)=\dfrac{x^3+3x^2-x^3-1-3x^2-3x}{x+3}=\dfrac{-3x-1}{x+3}$

$\rightarrow y=\dfrac{-3x-1}{x+3} \rightarrow x=\dfrac{-3y-1}{y+3} \rightarrow y=\dfrac{-3x-1}{x+3}=\dfrac{ax+b}{2x+d} \Big\} \begin{matrix} a=-6 \\ b=-2 \\ d=6 \end{matrix}$

$f^{-1}(b)=f^{-1}(-2)=\dfrac{-3(-2)-1}{-2+3}=5$

۱۰

$f(x)=\dfrac{x}{x^2+1} \rightarrow y(x^2+1)=x \rightarrow yx^2+y=x \rightarrow yx^2+y-x=0$

$\rightarrow x=\dfrac{-b\pm\sqrt{\Delta}}{2a} \quad a=y , b=-1 , c=y \rightarrow x=\dfrac{-(-1)\pm\sqrt{1-4y^2}}{2y}$

★ معکوس تابع در $[-\frac{1}{2},\frac{1}{2}]$ برقرار است

$f^{-1}(x)=\begin{cases} \dfrac{1+\sqrt{1-4x^2}}{2x} & x>0 \\ \dfrac{1-\sqrt{1-4x^2}}{2x} & x<0 \end{cases} \quad x\in[-\frac{1}{2},\frac{1}{2}]$