

$$f(x) = \begin{cases} x^3 - 4 & , x > a \rightarrow a^3 - 4 \\ 11x - 10 & , x \leq a \rightarrow 11a - 10 \end{cases}$$

$$f(x) \rightarrow R = (-\infty, 11a - 10]$$

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$$\Rightarrow a^3 - 4 \geq 11a - 10 \rightarrow a^3 - 11a + 6 \geq 0$$

$$\rightarrow (a-2)(a^2+2a-3) \geq 0 \rightarrow (a-2)(a+3)(a-1) \geq 0$$

$$\frac{-4 \pm 12}{-1 \pm 1} \rightarrow a \geq -2$$

$$a \geq -2$$

$$f(x) = 3x + k, f(2) = 4$$

$$f(v) = 11 - 10 = 1$$

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$$f^{-1}(x) = x \cdot 3y + k - 3y = x - k \rightarrow y = \frac{x-k}{3} = 4$$

$$k = -10$$

$$f(f(x))$$

$$f(f(x)) = 9x - 40$$

$$f(3x - 10) - 10 = 9x - 40 - 10$$

$$= 9x - 50$$

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$$f(x) = \frac{ax}{x-1} \rightarrow f^{-1}(x) = x = \frac{ya}{y-1} = y \cdot \frac{-x}{x-a} \cdot y = \frac{x}{x-a}$$

$$A(10, a) \Rightarrow \frac{10a}{10a - a} = a \rightarrow a = 1$$

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$$f \circ f^{-1} = \{(a, a), (v, v), (r, r), (x, x)\}$$

$$\begin{aligned} a &\rightarrow r \rightarrow a \\ v &\rightarrow x \rightarrow v \\ r &\rightarrow v \rightarrow r \\ x &\rightarrow a \rightarrow x \end{aligned}$$

$$\{(a, a), (r, r), (v, v), (x, x)\}$$

$$f^{-1} \circ f$$

$$\begin{aligned} r &\rightarrow a \rightarrow r \\ x &\rightarrow v \rightarrow x \\ v &\rightarrow r \rightarrow v \\ a &\rightarrow x \rightarrow a \end{aligned}$$

$$\{(r, r), (x, x), (v, v), (a, a)\}$$

$$f \circ g^{-1}$$

$$\begin{aligned} r &\rightarrow a \rightarrow a \\ x &\rightarrow v \rightarrow v \\ a &\rightarrow r \rightarrow r \\ v &\rightarrow x \rightarrow x \end{aligned}$$

$$\{(r, a), (a, v), (v, r), (v, x)\}$$

$$g^{-1} \circ f$$

$$\begin{aligned} r &\rightarrow a \rightarrow a \\ x &\rightarrow v \rightarrow v \\ v &\rightarrow r \rightarrow r \\ a &\rightarrow x \rightarrow x \end{aligned}$$

$$\{(r, a), (v, r), (a, v), (v, x)\}$$

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$$f(r, x) = (x, a) = (4, 0)$$

$$g = \{(r, 1), (x, 2), (v, 3)\}$$

$$h = \{(v, 2), (r, 3), (x, 4), (a, 0)\}$$

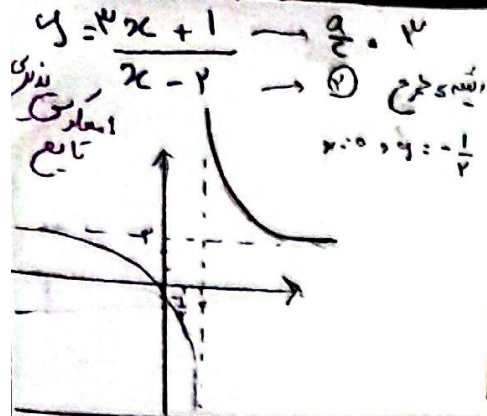
$$f \circ g^{-1} = \{(1, 4), (2, 3), (3, 0)\}$$

$$f(g^{-1}(x))$$

$$\begin{aligned} 1 &\rightarrow r \rightarrow r \\ 2 &\rightarrow x \rightarrow x \\ 3 &\rightarrow v \rightarrow v \end{aligned}$$

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$$h \circ f \circ g^{-1} = \{(1, \frac{1}{4}), (2, \frac{1}{3}), (3, \frac{1}{0})\} = \{(1, \frac{1}{4}), (2, \frac{1}{3})\}$$

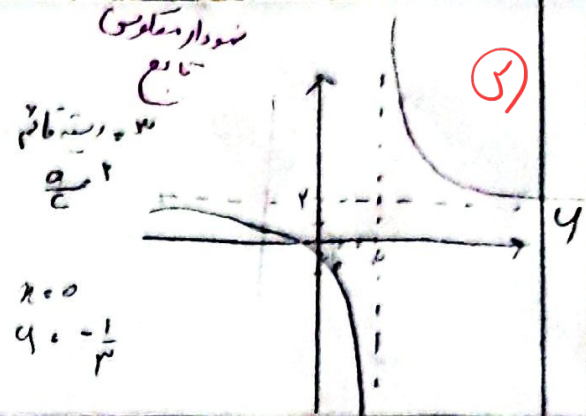


ضابطه
تابع

$$\frac{xy+1}{y-2} = x$$

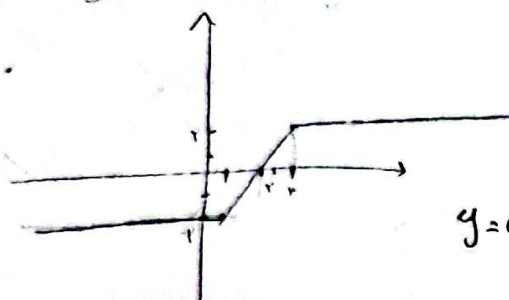
$$y = \frac{-2x-1}{x-2}$$

$$y = \frac{x+1}{x-2}$$



$f(x) = |x-1| - |x-3|$
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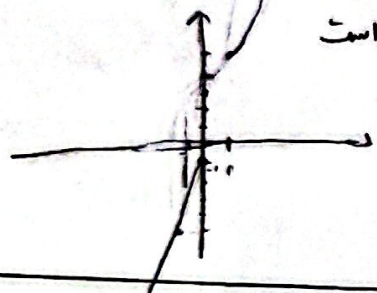
$y = 2x-1 \rightarrow x = \frac{y+1}{2}$
 $x \cdot 2y - f \rightarrow 2y = x + f$
 $y = \frac{x+f}{2}$
 $[-2, 2] \Rightarrow f^{-1}(x) = \frac{x+f}{2} \text{ for } x \in [-2, 2]$



$y = ax + b$
 $\begin{cases} 2a + b = 2 \\ -a + b = -2 \end{cases} \Rightarrow \begin{cases} a = 2 \\ b = -2 \end{cases}$
 $y = 2x - 1$

$f(x) = \begin{cases} x^2 + 1 & x \geq 1 \rightarrow R = [0, \infty) \\ 2x - 1 & x < 1 \rightarrow (-\infty, 1] \end{cases}$

$x = y^2 + 1 \rightarrow y^2 = x - 1 \rightarrow y = \sqrt{x-1}$
 $x = 2y - 1 \rightarrow 2y = x + 1 \rightarrow y = \frac{x+1}{2}$



$f^{-1}(x) = \begin{cases} \sqrt{x-1} & x \geq 1 \\ \frac{x+1}{2} & x < 1 \end{cases}$

$f(x) = x^3 - \frac{(x+1)^3}{2} \rightarrow f(x) = x^3 - \frac{x^3 + 3x^2 + 3x + 1}{2} = \frac{-x^3 - 3x^2 - 3x - 1}{2}$

$\rightarrow y = -\frac{3x-1}{2x+1} \rightarrow x = \frac{-2y-1}{3y+1} \rightarrow y = -\frac{3x-1}{2x+1}$

$f^{-1}(b) = f^{-1}(-1) = \frac{-3(-1)-1}{-2+1} = 2$

$f(x) = \frac{x}{x^2+1} \rightarrow y(x^2+1) = x \rightarrow yx^2 + y - x = 0$

$\rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \rightarrow a = y, b = -1, c = y \rightarrow x = \frac{-(-1) \pm \sqrt{1-4y^2}}{2y}$

$f^{-1}(x) = \begin{cases} \frac{1 + \sqrt{1-4x^2}}{2x} & x > 0 \\ \frac{1 - \sqrt{1-4x^2}}{2x} & x < 0 \end{cases}$
 $x \in [-\frac{1}{2}, \frac{1}{2}]$