با کیفیت بهتر عکس گرفته شود

۱۸,۷۵

①

$f(x)=\begin{cases} x^3-4 \;;\; x\geq a \to a^3-4 \\ 12x-20 \;;\; x<a \to 12a-20 \end{cases} \Rightarrow a^3-4 \geq 12a-20$

$a^3-12a+16\geq 0 \Rightarrow (a-2)^2(a+4)\geq 0$

$-\;|\;+\;|\;+$

$a\in[-4,+\infty)$

۱,۵

②

$f^{-1}(2)=4 \Rightarrow f(4)=2$

$f(x)=3x+k \Rightarrow f(4)=3(4)+k \Rightarrow k=-10 \Rightarrow f(x)=3x-10$

الف) $f(7)=3(7)-10=21-10=11$

ب) $f(f(x))=f(3x-10)=3(3x-10)-10=9x-30-10=9x-40$

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③

$f(x)=\frac{ax}{x-1} \xrightarrow{\text{وارون}} f\left(\frac{ax}{x-1}\right)=x \xrightarrow{A(2a,a)} f\left(\frac{a\cdot a}{a-1}\right)=2a \Rightarrow \frac{a^2}{a-1}=2a \Rightarrow 2a^2-2a=a^2$

$a^2-2a=0$

$a(a-2)=0 \to a=2$ ق ق

$\to a=0$ غ ق ← یعنی تابع ثابت می‌شود

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④

$g=\{(3,2),(8,4),(9,5)\}$ $f=\{(4,5),(6,7),(7,2),(9,7)\}$

$g^{-1}=\{(2,3),(4,8),(5,9)\}$ $f^{-1}=\{(5,4),(7,6),(2,7),(7,9)\}$

الف) $f\circ f^{-1}=f(f^{-1}(x))=\{(5,5),(7,7),(2,2),(7,7)\}$

$\to f^{-1}\circ f=f^{-1}(f(x))=\{(4,4),(6,6),(7,7),(9,9)\}$

ب) $f\circ g^{-1}=f(g^{-1}(x))=\{(3,5),(5,7)\}$

$\Rightarrow g^{-1}\circ f=g^{-1}(f(x))=\{(4,9),(7,3),(9,8)\}$

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⑤

$f=\{(2,4),(4,8),(5,0)\}$ $f(g^{-1}(x))=\{(1,4)(3,8)(5,0)\}$

$g=\{(2,1),(4,3),(5,5)\} \Rightarrow g^{-1}=\{(1,2),(3,4),(5,5)\}$ $\Rightarrow \frac{h}{f\circ g^{-1}}=\{(1,\frac{2}{4}),(3,\frac{6}{8})\}$

$h=\{(1,2),(3,6),(4,1),(5,1)\} \Rightarrow D_h=\{1,3,4,5\}$

$D_{f\circ g^{-1}}=\{1,3,5\}$ $\cap \to \{1,3,5\}$

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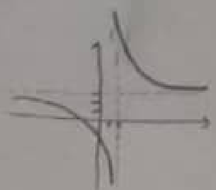
⑥



$y=\frac{3x+1}{x-2} \to$ هم‌گرافیک

$y=\frac{3x+1}{x-2} \Rightarrow yx-2y=3x+1$

$yx-3x=2y+1$

$x(y-3)=2y+1 \Rightarrow x=\frac{2y+1}{y-3} \Rightarrow y=\frac{2x+1}{x-3}$

انتقال معکوس

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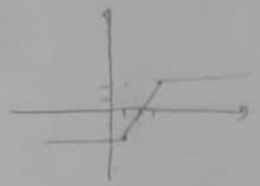
(۷)

$f(x) = |x-1| - |x-2|$ $\Rightarrow$ $x-1=0 \Rightarrow x=1$

$x-2=0 \Rightarrow x=2$



$\Rightarrow f(x) = \begin{cases} -1 & x \le 1 \\ 2x-3 & 1 < x < 2 \\ 1 & x \ge 2 \end{cases}$ $\Rightarrow$ $2x-3 = y \Rightarrow 2x = y+3 \Rightarrow x = \frac{y}{2}+\frac{3}{2} \Rightarrow y = \frac{x}{2}+\frac{3}{2}$

(۵)

(۸)

$f(x) = \begin{cases} x^3+1 & x \ge 1 \\ 2x-1 & x \le 0 \end{cases}$ $\begin{matrix} \longrightarrow R_f = [0,+\infty) \\ \longrightarrow R_f = (-\infty,-1] \end{matrix}$ $\Rightarrow$ هیچ اشتراکی ندارند $\Rightarrow$ یک‌به‌یک است

$y = x^3+1 \Rightarrow x = \sqrt[3]{y-1} \Rightarrow y = \sqrt[3]{x-1}$

$y = 2x-1 \Rightarrow x = \frac{1+y}{2} \Rightarrow y = \frac{1+x}{2}$

(۵)

$f^{-1}(x) = \begin{cases} \sqrt[3]{x-1} & x \ge 0 \\ \frac{x+1}{2} & x \le -1 \end{cases}$

(۹)

$f(x) = x^2 - \frac{(x+1)^3}{x+2}$, $f^{-1}(x) = \frac{ax+b}{2x+d}$

$\frac{x^2(x+2)-(x+1)^3}{x+2} = \frac{-3x-1}{x+2}$ $\xrightarrow{\text{وارون}}$ $y = \frac{-3x-1}{x+2} \Rightarrow yx+2y = -3x-1$

$yx+3x = -1-2y$

$x(y+3) = -1-2y \Rightarrow x = \frac{-1-2y}{y+3} \Rightarrow y = \frac{-1-2x}{x+3}$

$d=3$, $b=-2$, $a=-1$ $\Leftarrow$ مقایسه $y = \frac{-2-3x}{2x+3}$

(۱,۲۵)

$f^{-1}(b) = f^{-1}(-2) = \frac{-4(-2)-2}{2(-2)+3} = \frac{14-2}{-4+3} = -10$

$\frac{10}{2} = 5$

(۱۰)

$f(x) = \frac{x}{x^2+1} \Rightarrow y = \frac{x}{x^2+1} \Rightarrow yx^2+y = x \Rightarrow yx^2-x+y=0 \Rightarrow x = \frac{1 \pm \sqrt{1-4y^2}}{2y}$

$\Rightarrow y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$, $x \neq 0$

$b^2-4ac = 1-(4)(y)(y) = 1-4y^2$

(۲)