مقطع: یازدهم دفتر ۸

۱۸٫۵

(۱)

$$f(x) = \begin{cases} x^3 - 4 & x \geq a \\ 12x - 20 & x < a \end{cases}$$

$12a - 20 \leq a^3 - 4 \xrightarrow{} a^3 - 12a + 16 \geq 0 \longrightarrow \underline{a \geq -4}$

(۳)

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(۲)

$f^{-1}(2) = 4 \qquad f(x) = 3x + k$ (-10) $\qquad f(4) = 2 \rightarrow f(4) = 3 \times 12 + k \rightarrow \boxed{K = -10}$

(۵)

الف) $f(7) = ? \qquad f(7) = 21 - 10 = 11$

ب) $f(f(x)) = ? \qquad f(3x - 10) \Rightarrow 3(3x - 10) - 10 \Rightarrow \boxed{9x - 40}$

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(۳)

$A = (2a, a)$ (x, y) $\qquad f(x) = \dfrac{ax}{x-1} \Rightarrow 2\cancel{a} = \dfrac{\cancel{2a}\,a}{a-1} \rightarrow 2a - 2 = a$

$\boxed{a = 2}$

(۳)

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(۴)

$f = \{(3,5)(4,7)(7,2)(9,6)\}$ ($5,3 \quad 7,4 \quad 2,7 \quad 6,9$) $\qquad g = \{(3,2)(8,6)(9,5)\}$ ($2,3 \quad 6,8 \quad 5,9$)

الف) $f \circ f^{-1} \qquad f(f^{-1}(x)) \Rightarrow \{(5,5)(7,7)(2,2)(6,6)\}$

(۳)

ب) $f^{-1} \circ f = (3,3)(4,4)(7,7)(9,9) \qquad 3 \rightarrow 5 \rightarrow 3$

ج) $f \circ g^{-1} \quad \{(2,5)(5,6)\}$

د) $g^{-1} \circ f \Rightarrow \{(3,9)(9,8)(7,3)\}$

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(۵)

$f = \{(2,4)(4,8)(6,0)\} \qquad g = \{(2,1)(4,3)(6,5)\}$ ($1,2 \quad 3,4 \quad 5,6$) $\qquad h = \{(1,2)(3,4)(4,1)\boxed{(5,1)}\}$

$\dfrac{h}{f \circ g^{-1}} = ? \quad \{\tfrac{1}{4}\} \quad \{(1,\tfrac{1}{2})(3,\tfrac{1}{2})\cancel{(5,\tfrac{1}{0})}\}$

(۱٫۲۵)

$f \circ g^{-1} \Rightarrow 1 \rightarrow 2 \rightarrow 4 \qquad 3 \rightarrow 4 \rightarrow 8 \qquad 5 \rightarrow 6 \rightarrow 0 \qquad (1,4)\ (3,8)\ \boxed{(5,0)}$

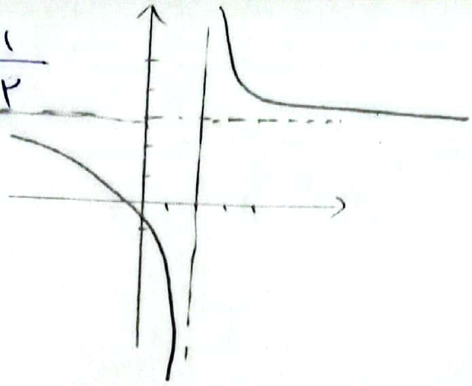
۲

یک به یک است.

$y=\frac{3x+1}{x-2}$

مجانب قائم $=2$

مجانب افقی $=3$



$y=\frac{3x+1}{x-2} \xrightarrow{\text{معکوس}} x=\frac{3y+1}{y-2}$

$xy-2x=3y+1$

$-2x-1=y(3-x) \rightarrow \boxed{y=\frac{2x+1}{-3+x}}$

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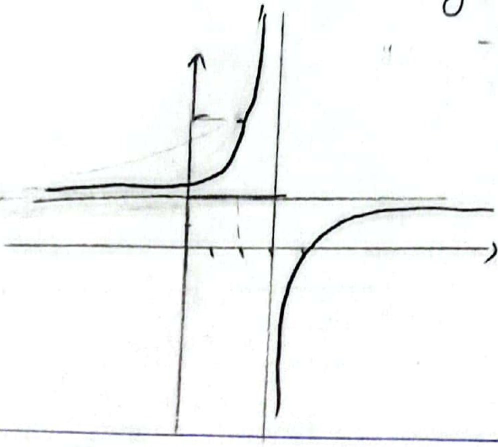
$y=\frac{2x+1}{x-3}$

مجانب قائم $=3$

مجانب افقی $=2$

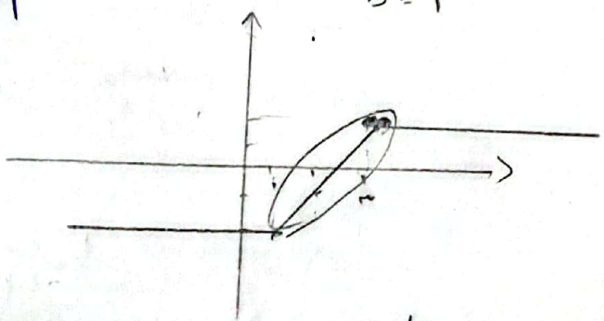


۳

$f(x)=|x-1|-|x-3|$

$$\begin{cases} -2 & x<1 \\ 2x-4 & 1\leq x\leq 3 \\ 2 & x>3 \end{cases}$$

$a=1 \quad b=3$



$y=2x-4$

$x=2y-4 \xrightarrow{\text{معکوس}} \boxed{\frac{x+4}{2}=y}$

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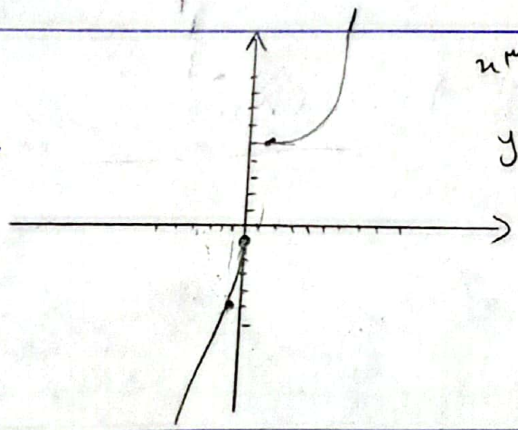
۱

$f(x)=\begin{cases} x^3+4 & x\geq 1 \\ 4x-1 & x\leq 0 \end{cases}$



$x^3+4=y \rightarrow y^3+4=x \rightarrow y=\sqrt[3]{x-4} \quad x\geq 5$

$y=4x-1 \rightarrow x=4y-1 \rightarrow \frac{x+1}{4}=y \quad x\leq -1$

۱,۷۵

۴

$f(x)=x^2-\frac{(x+1)^3}{x+3} \rightarrow f(x)=\frac{\cancel{x^3}+\cancel{3x^2}-\cancel{x^3}-1-3x-\cancel{3x^2}}{x+3} \rightarrow f(x)=\frac{3x+1}{-x-3}$

$\xrightarrow{\text{معکوس}} x=\frac{3y+1}{-y-3} \rightarrow -xy-3x=3y+1$

$-3x-1 \Rightarrow y(3+x) \Rightarrow y=\frac{-3x-1}{3+x} \xrightarrow{\times 2} y=\frac{-6x-2}{6+2x}$

۳

$f^{-1}(b)=? \rightarrow -2=\frac{3x+1}{-x-3} \rightarrow 2x+6=3x+1$

$\boxed{x=5}$

$a=-6 \quad b=-2$

$d=6$

۱,۵

۵

$f(x)=\frac{x}{x^2+1} \rightarrow \underset{a}{y}x^2 \underset{b}{-x} + \underset{c}{y}=$

$x \rightarrow \frac{\sqrt{1-4y^2}\pm 1}{2y}$

$1-4y^2\geq 0 \rightarrow \Delta\geq 0, |a|$

$\boxed{f^{-1} \Rightarrow \frac{\sqrt{1-4x^2}\pm 1}{2x}}$

$\frac{1\pm\sqrt{1-4y^2}}{2y} \rightarrow$