سارا سامانی　　تکلیف شماره ۱۱　　یازدهم دختر ۹

۱-

$f(x)=\begin{cases} x^3-4 & ; x> a \\ 12x-20 & ; x\le a\end{cases} \Rightarrow x^3-4=12x-20 \to x^3-12x+16=0 \to a^3-12a+16=0 \to (a-2)^2(a+4)\ge 0$ (فاکتور)

$\Rightarrow a\ge -4$

۲-

$f^{-1}(2)=4 \Rightarrow f(4)=2 \qquad f(4)=4\times 3+K \Rightarrow 12+K=2\to K=-10$

$f(7)=(3\times 7)-10=11$

$f(f(x))=3(3x+K)+K=9x+4K=9x-40$

۳-

$f^{-1}(x)=\frac{ay}{y-1} \Rightarrow x=\frac{ay}{y-1} \Rightarrow \frac{ay}{y-1}=2a \to ay=2ay-2a \to 2a=ay \to y=2 \Rightarrow a=2$

۴-

$f=\{(3,5)(4,7)(7,2)(9,6)\} \qquad g=\{(3,2)(8,9),(9,5)\} \qquad f^{-1}=\{(5,3)(7,4)(2,7)(6,9)\}$

$g^{-1}=\{(2,3)(9,8)(5,9)\}$

الف) $f\circ f^{-1}=\{(5,5),(7,7)(2,2)(6,6)\}$

ب) $f^{-1}\circ f=\{(3,3)(4,4)(7,7)(9,9)\}$

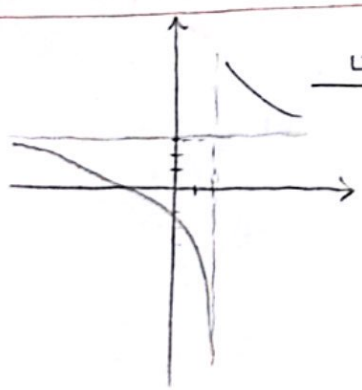
ج) $f\circ g^{-1}=\{(2,5)(5,6)\}$

د) $g^{-1}\circ f=\{(3,9)(7,3)(9,8)\}$

۵-

$g^{-1}=\{(1,2)(3,4)(5,6)\} \longrightarrow f\circ g^{-1}=\{(1,4)(3,8)(5,0)\} \longrightarrow \frac{h}{f\circ g^{-1}}=\{(1,\frac{1}{2})(3,\frac{1}{2})~~(5,\frac{1}{0})\}$ غ ق ق

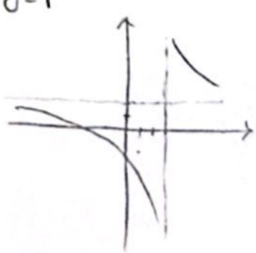
۶-



یک به یک است ⇐ معکوس پذیر است

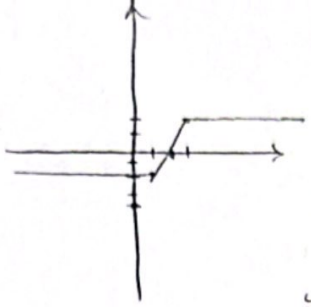
$\to f^{-1} \Rightarrow x=\frac{3y+1}{y-2} \longrightarrow xy-2x=3y+1 \longrightarrow$

$y(x-3)=2x+1 \to y=\frac{2x+1}{x-3}$



۸-

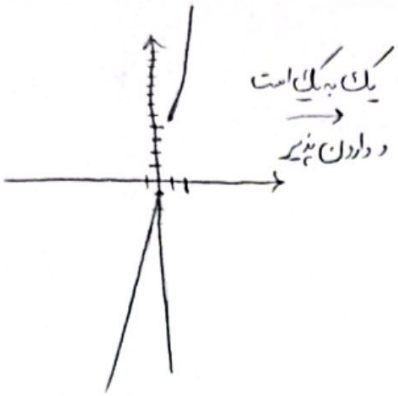
$$f(x)\begin{cases}2 & x\geq 2\\ 2x-4 & 1<x<2\\ -2 & x\leq 1\end{cases}$$



در بازه $[1,2]$ ⟸ یک به یک و وارون پذیر است

$y=2x-4 \Longrightarrow x=\frac{y+4}{2} \rightarrow x+4=2y \rightarrow y=\frac{x+4}{2}$

$$f(x)=\begin{cases}x^2+4 & ;x\geq 1\\ 2x-1 & ;x<0\end{cases}$$



۷- $y=x^2+4 \rightarrow x\geq 1 \rightarrow R_f=[5,+\infty)$

$x=y^2+4 \rightarrow y^2\geq 0$

$y^2=x-4 \rightarrow y=\sqrt[2]{x-4}$, $x\geq 5$

$y=2x-1 \rightarrow x<0 \rightarrow R_f=(-\infty,-1]$

$x=\frac{y+1}{2} \rightarrow 2x-1=y \rightarrow 2y=x+1 \rightarrow$

$y=\frac{x+1}{2}$, $x\leq -1$

$$f^{-1}(x)=\begin{cases}\sqrt{x-4} & ;x\geq 5\\ \frac{x+1}{2} & ;x\leq -1\end{cases}$$

۹-

$$f(x)=\frac{x^2-(x+1)^3}{x+2}=\frac{x^3+3x^2+3x+1-x^3-3x^2-3x-1}{x+2}=\frac{-2x-1}{x+2} \rightarrow x=\frac{-2y-1}{y+2} \Rightarrow xy+2x=-2y-1 \rightarrow$$

$$y(x+2)=-1-2x \rightarrow y=\frac{-1-2x}{x+2} \rightarrow y=\frac{-2x-1}{x+2}\times\frac{2}{2}=\frac{-4x-2}{2x+4} \rightarrow \begin{matrix}a=-4\\ b=-2\\ d=4\end{matrix}$$

$$f^{-1}(b)=f^{-1}(-2) \rightarrow f^{-1}(-2)=\frac{4+(-2)(-2)}{2(-2)+4}=\frac{8}{0}$$

۱۰-

$$y=\frac{x}{2x^2+1} \longrightarrow y(2x^2+1)=x \Longrightarrow 2yx^2-x+y=0 \rightarrow x=\frac{1\pm\sqrt{1-4y^2}}{4y} \Rightarrow f^{-1}(x)=\frac{2x}{1+\sqrt{1-4x^2}},\ x\in\left[-\frac{1}{4},\frac{1}{4}\right]$$