سارا سامانی — تکلیف شماره ۱۱ — یازدهم دختر ۹

۱۹٫۵

۱-

$$f(x)=\begin{cases}x^3-4 & ; x>a\\ 12x-20 & ; x\le a\end{cases} \Rightarrow x^3-4=12x-20 \rightarrow x^3-12x+16=0 \rightarrow a^3-12a+16=0 \rightarrow (a-2)^2(a+4)\ge 0$$

فاکتور

$$\Rightarrow a \ge -4$$

۲-

$$f^{-1}(2)=4 \Rightarrow f(4)=2 \qquad f(4)=4\times 3+K \Rightarrow 12+K=2 \rightarrow K=-10$$

$$f(7)=(3\times 7)-10=11$$

$$f(f(x))=3(3x+K)+K=9x+4K=9x-40$$

۳-

$$f^{-1}(x)=\frac{ay}{y-1} \Rightarrow x=\frac{ay}{y-1} \Rightarrow \frac{ay}{y-1}=2a \rightarrow ay=2ay-2a \rightarrow 2a=ay \rightarrow y=2 \Rightarrow a=2$$

۴-

$$f=\{(3,5)(4,7)(7,2)(9,6)\} \quad g=\{(3,2)(8,9),(9,5)\} \quad f^{-1}=\{(5,3)(7,4)(2,7)(6,9)\}$$

$$g^{-1}=\{(2,3)(9,8)(5,9)\}$$

الف) $f\circ f^{-1}=\{(5,5),(7,7)(2,2)(6,6)\}$

ب) $f^{-1}\circ f=\{(3,3)(4,4)(7,7)(9,9)\}$

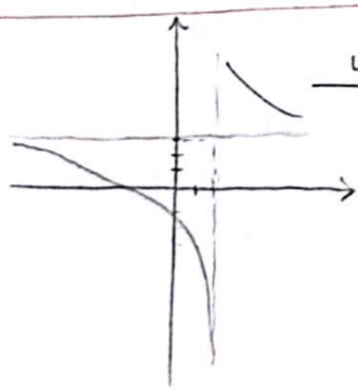
ج) $f\circ g^{-1}=\{(2,5)(5,6)\}$

د) $g^{-1}\circ f=\{(3,9)(7,3)(9,8)\}$

۵-

$$g^{-1}=\{(1,2)(3,4)(5,6)\} \longrightarrow f\circ g^{-1}=\{(1,4)(3,8)(5,0)\} \longrightarrow \frac{x}{f\circ g^{-1}}=\{(1,\tfrac{1}{4})(3,\tfrac{1}{2})~~(5,\tfrac{5}{0})~~\}$$

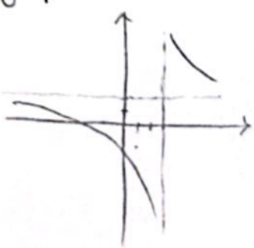
۶-



یک به یک است ⇐ معکوس پذیر است

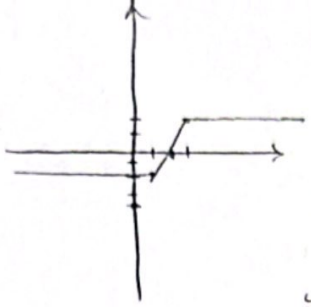
$$\rightarrow f^{-1} \Rightarrow x=\frac{3y+1}{y-2} \longrightarrow xy-2x=3y+1 \longrightarrow$$

$$y(x-3)=2x+1 \rightarrow y=\frac{2x+1}{x-3}$$



۸-

$$f(x)\begin{cases}2 & x\geq 2\\ 2x-4 & 1<x<2\\ -2 & x\leq 1\end{cases}$$

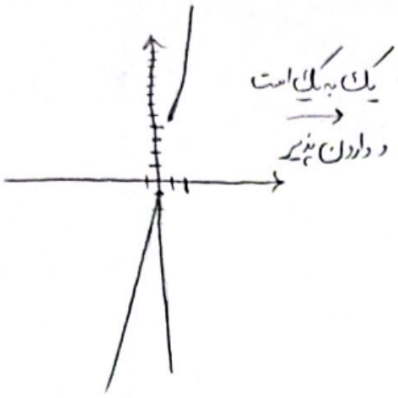


در بازه $[1,2]$ ⟸ یک به یک و وارون پذیر است

(۵)

$y=2x-4 \Rightarrow x=\frac{y}{2}-2 \rightarrow x+2=\frac{y}{2} \rightarrow y=\frac{x+4}{2}$

$$f(x)=\begin{cases}x^2+4 & ;x\geq 1\\ 2x-1 & ;x<0\end{cases}$$



$y=x^2+4 \rightarrow x\geq 1 \rightarrow R_f=[5,+\infty)$ ۷-

$x=y^2+4 \rightarrow y\geq 0$

$y^2=x-4 \rightarrow y=\sqrt{x-4}$, $x\geq 5$ (۵)

$y=2x-1 \rightarrow x<0 \rightarrow R_f=(-\infty,-1]$

$x=\frac{y}{2}-1 \rightarrow y\leq -1 \rightarrow \frac{y}{2}=x+1 \rightarrow$

$y=\frac{x+1}{2}$, $y\leq -1$

$$f^{-1}(x)=\begin{cases}\sqrt{x-4} & ;x\geq 5\\ \frac{x+1}{2} & ;x\leq -1\end{cases}$$

۹-

$$f(x)=\frac{x^2-(x+1)^3}{x+2}=\frac{x^3+3x^2-x^3-3x^2-3x-1}{x+2}=\frac{-3x-1}{x+2} \rightarrow x=\frac{-2y-1}{y+3} \Rightarrow xy+3x=-2y-1 \rightarrow$$

$$y(x+2)=-1-3x \rightarrow y=\frac{-1-3x}{x+2} \rightarrow y=\frac{-3x-1}{x+2}\times\frac{2}{2}=\frac{-6x-2}{2x+4} \rightarrow \begin{matrix}a=-6\\ b=-2\\ d=4\end{matrix}$$

(۵)

$f^{-1}(b)=f^{-1}(4) \rightarrow$

$$f^{-1}(4)=\frac{8+(-2)}{2(-2)+4}=\frac{6}{0}$$

۱۰- (۱٫۵)

$$y=\frac{x}{2x^2+1} \rightarrow y(2x^2+1)=x \Rightarrow 2yx^2-x+y=0 \rightarrow x=\frac{1\pm\sqrt{1-4y^2}}{4y} \Rightarrow f^{-1}(x)=\frac{1+\sqrt{1-4x^2}}{4x},\ x\in\left[-\frac{1}{4},\frac{1}{4}\right]$$

$$\frac{1\pm\sqrt{1-4x^2}}{4x}$$