پیتابزم یازدهم A

①

$12a-20 \leq a^3-4 \to a^3-12a+16 \geq 0 \to (a-2)^2(a+4) \geq 0$

x		-4		2	
P	$-$	0	$+$	0	$+$

$[-4,+\infty)$

②

$f^{-1}(x) \Rightarrow x = 3y+k \to 3y = -k+x \to y = \frac{x-k}{3}$

$f^{-1}(2) = \frac{2-k}{3} = 4 \to \boxed{k=-10}$

الف) $f(7) = 21-10 = 11$ ب) $fof(x) = 3(3x-10)-10 = 9x-40$

③

$f^{-1}(x) \Rightarrow x = \frac{ay}{y-1} \to xy - x = ay \to y = \frac{x}{x-a} \xrightarrow{\left|{}^{2a}_{a}\right.}$

$\to a = \frac{2a}{2a-a} \to a = \frac{2a}{a} \to a^2-2a=0 \quad a \to 2,\ a \to 0$

④

$f^{-1} = \{(5,3)(7,4)(2,7)(6,9)\} \quad g^{-1} = \{(2,3)(6,8)(5,9)\}$

الف)
$5 \to 3 \to 5$
$7 \to 4 \to 7$
$2 \to 7 \to 2$
$6 \to 9 \to 6$
$\{(5,5)(7,7)(2,2)(6,6)\}$

ب)
$3 \to 5 \to 3$
$4 \to 7 \to 4$
$7 \to 2 \to 7$
$9 \to 6 \to 9$
$\{(3,3)(4,4)(7,7)(9,9)\}$

ج)
$2 \to 3 \to 5$
$6 \to 8 \to \times$
$5 \to 9 \to 6$
$\{(2,5)(5,6)\}$

د)
$3 \to 5 \to 9$
$4 \to 7 \to \times$
$7 \to 2 \to 3$
$9 \to 6 \to 8$
$\{(3,9)(7,3)(9,8)\}$

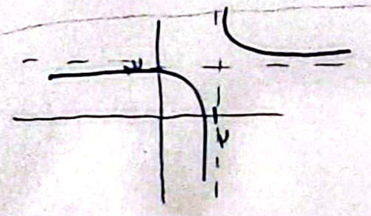
⑤

$fog^{-1} = \{(1,4)(3,8)(5,0)\}$

$h = \{(1,2)(3,2)(4,1)(5,1)\}$

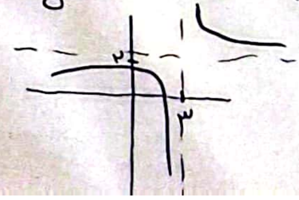
$\frac{h}{fog^{-1}} = \{(1,\frac{2}{4}),(3,\frac{2}{8})\} = \{(1,\frac{1}{2}),(3,\frac{1}{4})\}$

⑥

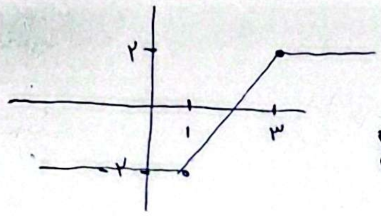


یک‌به‌یکه ✓ $\to f^{-1}(x) \Rightarrow x = \frac{3y+1}{y-2} \to y = \frac{2x+1}{x-3}$

شکل تابع وارون



(7)



$[a, b] = [1, 3]$ ضابطه قسمت معکوس: $2 = 3a + b$

$-2 = a + b$

$a = 2 \quad b = -4$

$y = 2x - 4 \xrightarrow{\text{معکوس}} x = 2y - 4 \to y = \dfrac{x+4}{2}$

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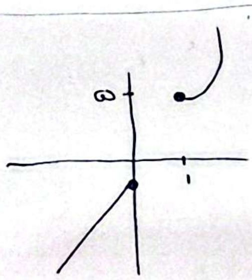
(8)



برد تابع $f(x)$ برابر با دامنه $f^{-1}(x)$

$$f^{-1}(x) \begin{cases} \sqrt[3]{x-4} & x \geq 5 \\ \dfrac{x+1}{4} & x \leq -1 \end{cases}$$

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(9)

$$f(x) = \frac{x^3 + 3x^2 - (x^3 + 1 + 3x^2 + 3x)}{x+3} = \frac{3x+1}{-3-x} \xrightarrow{f^{-1}(x)} x = \frac{3y+1}{-3-y}$$

$$y = \frac{3x+1}{-x-3} \to \frac{-6x-2}{2x+6} = \frac{ax+b}{2x+d}$$

$a = -6$

$b = -2$

$d = 6$

$f^{-1}(2) = \dfrac{-12+2}{4+6} = -1/4$

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(10)

$$x = \frac{y}{y^2+1} \to xy^2 + x = y \to xy^2 + x - y = 0 \qquad y = \frac{1 \pm \sqrt{1-4x^2}}{2x}$$

(I) $1 - 4x^2 \geq 0 \to 1 \geq 4x^2 \to \frac{1}{4} \geq x^2 \to \left[-\frac{1}{2}, \frac{1}{2}\right]$

$$f^{-1}(x) \begin{cases} \dfrac{1-\sqrt{1-4x^2}}{2x} & 0 < x \leq \dfrac{1}{2} \\ \dfrac{1+\sqrt{1-4x^2}}{2x} & -\dfrac{1}{2} \leq x < 0 \end{cases}$$