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الف) $f = \{(3, 2), (n, 2), (3, n^2 - n), (m, 2), (-1, 2)\}$
 $n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow n = -1 \rightarrow \text{غیر قابل قبول} \rightarrow (-1, 2)$
 $(3, 2)(m, 2) \rightarrow m = 3$

ب) $f = \{(-1, 1), (1, 2), (2, 3), (a, m-1), (a+2, n), (m, 2)\}$

$(3, 2)(m, 2) \rightarrow m = 3 \rightarrow (a, m-1) = (a, 1)$
 $(-1, 1)(a, 1) \rightarrow a = -1 \rightarrow (1, n) \rightarrow n = 2$

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$f(x) = \begin{cases} x_{n-1} & ; x \geq 1 \\ x+a & ; x < 1 \end{cases} \quad a = ? \rightarrow a \leq 1$

$x_{n-1} = x_1 + a$ if $x_1 \geq 1 \rightarrow x_1 = x_{n-1} - a \rightarrow x_{n-1} - a < 1 \rightarrow x_{n-1} - a - 2 < 0$
 $\rightarrow x_1 - a - 2 = 1 - a \rightarrow a \leq 1$ قابل قبول

-3

$f(x) = \begin{cases} x_{n-1} & ; x \geq 1 \\ ax+a-1 & ; x \leq 0 \end{cases} \quad a = ? \rightarrow 0 \leq a < 3$

$ax_1 + a - 1 = x_{n-1} - 1 \rightarrow ax_1 + a = x_{n-1} \rightarrow x_1 = \frac{x_{n-1} - a}{a} \rightarrow a \neq 0$

$\rightarrow x_1 \leq 0 \rightarrow \frac{x_{n-1} - a}{a} \leq 0 \rightarrow a > 0, x_{n-1} - a \leq 0 \rightarrow a \geq x_{n-1} \rightarrow$ غیر قابل قبول

$\rightarrow 0 < a < 3, x_{n-1} - a > 0 \rightarrow$ قابل قبول

$\rightarrow a < 0, x_{n-1} - a \geq 0 \rightarrow$ غیر قابل قبول

$\rightarrow a = 0 \rightarrow \text{غیر قابل قبول} \rightarrow f(x) = -1 \rightarrow a = 0 \rightarrow$ قابل قبول

الف) $y = x^3 + 2 \rightarrow x_1 < x_2 \rightarrow x_2^3 - x_1^3 = (x_2 - x_1)(x_2^2 + x_1x_2 + x_1^2)$

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$\rightarrow y = x^3 + 2 \rightarrow x = \sqrt[3]{y-2}$

درست است

ب) $y = x^3 - 2x + 2$

$f(x) = (x-2)^3 - 2 \rightarrow f(t+1) = (t)^3 - 2 = t^3 - 2$

درست است

$(x-2)^3 = y+2 \rightarrow x = \sqrt[3]{y+2} + 2$ if $x \geq 2$ $\xrightarrow{\text{حقیقی}} y+2 \geq 0 \rightarrow y \geq -2$
 $\rightarrow x = 2 - \sqrt[3]{y+2}$ if $x \leq 2$

معمولاً ۱۰

۵- $y = \frac{3n+1}{n-2} \rightarrow \text{if } f(n_1) = f(n_2) \rightarrow \frac{3n_1+1}{n_1-2} = \frac{3n_2+1}{n_2-2}$

$(3n_1+1)(n_2-2) = (3n_2+1)(n_1-2) \rightarrow 3n_1n_2 - 6n_1 + n_2 - 2 = 3n_1n_2 - 6n_2 + n_1 - 2$

$\rightarrow -6n_1 + n_2 = -6n_2 + n_1 \rightarrow -7n_1 + 7n_2 = 0 \rightarrow n_1 = n_2 \rightarrow \text{یک به یک}$

ب) $y = \frac{2n}{n+2}$ معکوس $= \frac{2y+1}{y-2} \quad y \neq 2$

$2n = 2(n+2) \rightarrow n \neq -2 \rightarrow \frac{2(n+2)}{n+2} = 2 \rightarrow \text{یک به یک نیست}$

۶- $y = \sqrt{x-3} \rightarrow \text{یک به یک} \rightarrow \text{تابع صعودی}$

$x-3 \geq 0 \rightarrow x \geq 3$

معکوس $\rightarrow y^2 + 3, y \geq 0$

ب) $y = x^2 - 8x + 3, x \in [2, +\infty)$

$x^2 - 8x + 3 = (x^2 - 8x + 8) - 5 = (x-4)^2 - 5$ if $x \geq 2 \rightarrow \text{تابع صعودی} \rightarrow \text{یک به یک}$

معکوس $\rightarrow 4 + \sqrt{y+5}, y \geq -5$

۷- $f(x) = x + \sqrt{x} \in (0, +\infty) \rightarrow f^{-1}(x) = ax + b\sqrt{x} - c$

$a + 2b + 8c = ?$

$y = f(x) = x + \sqrt{x} \rightarrow x = f^{-1}(y) \rightarrow y = t^2 + t \rightarrow t^2 + t - y = 0$

$t = \frac{-1 + \sqrt{1+4y}}{2}, t > 0 \rightarrow x = t^2 = \left(\frac{-1 + \sqrt{1+4y}}{2}\right)^2 \rightarrow \text{معکوس} = x + \frac{1}{x} - \frac{\sqrt{1+4x}}{2}$

$f(x) = ax + b\sqrt{x} - c$

$f^{-1}(x) = x + \frac{1}{x} - \frac{1}{2}\sqrt{1+4x}$

$\left. \begin{array}{l} a=1, b=\frac{1}{x}, c=-\frac{1}{2} \end{array} \right\} \Rightarrow a + 2b + 8c = 1 + 1 - 1 = 1$

۸- $y = \frac{x}{\sqrt{x^2-1}} \rightarrow x^2-1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1 \text{ و } x < -1 \rightarrow$

$D = (-\infty, -1) \cup (1, +\infty)$

$\left. \begin{array}{l} \text{if } x > 1 \rightarrow \text{تابع نزولی} \rightarrow \text{یک به یک} \\ \text{if } x < -1 \rightarrow \text{تابع نزولی} \rightarrow \text{یک به یک} \end{array} \right\} \text{در کل تابع یک به یک}$

معکوس $\rightarrow \sqrt{\frac{xy^2}{y^2-1}}, y > 1$

$\rightarrow -\sqrt{\frac{xy^2}{y^2-1}}, y < -1$

مسئله ۱۰

اصول ریاضی

-۱۰

$$f(x) = \sqrt[3]{x-1}, \quad g(x) = f(x) + \sqrt{f(x)} \quad g^{-1}(15) \rightarrow \begin{matrix} 3 \\ -8 \end{matrix}$$

$$y = \sqrt[3]{x-1} \rightarrow x-1 = y^3 \rightarrow x = y^3 + 1 \rightarrow f(x) = x^3 + 1$$

$$g(x) = f(x) + \sqrt{f(x)} = (x^3 + 1) + \sqrt{x^3 + 1}$$

$$g(a) = 15 \rightarrow a^3 + 1 + \sqrt{a^3 + 1} = 15 \rightarrow t + \sqrt{t} = 15 \rightarrow t = u^2$$

$$\rightarrow u^2 + u = 15 \rightarrow u^2 + u - 15 = 0 \rightarrow u = \frac{-1 \pm \sqrt{1 + 60}}{2} = \frac{-1 \pm \sqrt{61}}{2}$$

$$u_1 = 3 \quad u_2 = -8$$

$$f^{-1}(x) = \frac{x}{1+|x|}, \quad g(x) = \sqrt{x} - 1 \rightarrow y+1 = \sqrt{x} \rightarrow x = (y+1)^2$$

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$$f\left(-\frac{r}{d}\right) + g^{-1}\left(-\frac{r}{d}\right) \rightarrow g^{-1}\left(-\frac{r}{d}\right) = \left(-\frac{r}{d} + 1\right)^2 = \left(\frac{r}{d}\right)^2 = \frac{9}{10}$$

$$\frac{t}{1+|t|} = -\frac{r}{d} \rightarrow \text{if } t \geq 0 \rightarrow \frac{t}{1+t} \geq 0 \neq -\frac{r}{d}$$

$$\rightarrow \text{if } t < 0 \rightarrow |t| = -t \rightarrow \frac{t}{1-t} = -\frac{r}{d} \rightarrow f\left(-\frac{r}{d}\right) = -\frac{r}{d}$$

$$\text{پس } \Rightarrow -\frac{r}{d} + \frac{9}{10} = \frac{-20 + 9\sqrt{10}}{10} = \left(-\frac{20}{10} + \frac{9\sqrt{10}}{10}\right)$$