آکادمی | پاسخنامه تشریحی شماره ۱۰ | یازدهم ریاضی دختر A | 19,75

۱)
الف) $f=\{(3,2),(x,5),(3,x^2-x),(m,2),(-1,4)\}$

$2=x^2-x \Rightarrow 2=x(x-1) \Rightarrow x=2$ ، -1 غ ق ق

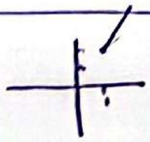
$m=3$

ب) $f=\{(-1,1),(1,2),(2,3),(a,m-1),(a+2,x),(m,3)\}$

$m=2 \qquad m-1=1 \Rightarrow a=-1 \qquad a+2=1 \Rightarrow x=2$

3

۲)
$$f(x)=\begin{cases}3x-1 & ; x\geq 1\\ x+a & ; x<1\end{cases}$$



$x\geq1, 3x-1, R=[2,+\infty)$

$x+a<2 \overset{x=1}{\Longrightarrow} 1+a<2 \Rightarrow a\leq 1 \Rightarrow a=(-\infty,1]$

$x=1 \to y=2$ ، $(3x-1)\cap(x+a)=\varnothing \Rightarrow$ یک به یک بودن

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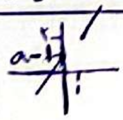
۳)
$$f(x)\begin{cases}3x-1 & ; x\geq 1\\ ax+a-1 & ; x\leq 0\end{cases}$$



$x\geq1 \quad y=[2,+\infty)$ ، $0\geq x \quad y=(-\infty,a-1]$ ، $\cap=\varnothing$ ، $0<a$ ، $\Rightarrow a-1<2 \Rightarrow a<3$ ، $\cap=0<a<3$

منفی هم نمی‌شود

شیب صفر شود تابع یک به یک نمی‌شود $a\neq 0 \Rightarrow$

$x=1 \Rightarrow 3-1=2 \qquad x=0 \Rightarrow a-1$

3

۴)
الف) $y=x^3+2 \Rightarrow x_1^3+\cancel{2}=x_2^3+\cancel{2} \Rightarrow x_1^3=x_2^3 \Rightarrow x_1=x_2 \to$ یک به یک است

معکوس: $x=y^3+2 \Rightarrow y^3=x-2 \Rightarrow y=\sqrt[3]{x-2} \Rightarrow f^{-1}(x)=\sqrt[3]{x-2}$

ب) $y=x^2-4x+2 \Rightarrow$ نمودار سهمی $\Rightarrow$ یک به یک نیست

3

۵)
الف) $y=\frac{3x+1}{x-2} \Rightarrow$ هموگرافیک $\to$ یک به یک

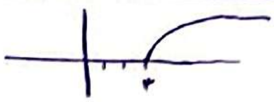
معکوس: $x=\frac{3y+1}{y-2} \Rightarrow yx-2x=3y+1 \Rightarrow -1-2x=3y-yx \Rightarrow -1-2x=y(3-x) \Rightarrow$

$y=\frac{-1-2x}{3-x} \Rightarrow y=\frac{2x+1}{x-3} \Rightarrow f^{-1}(x)=\frac{2x+1}{x-3}$

ب) $y=\frac{4+2x}{x+2} \Rightarrow$ تابع ثابت $2 \to$ یک به یک نیست

2

۶)
الف) $y=\sqrt{x-3} \Rightarrow$



$\Rightarrow$ یک به یک

$x-3\geq0 \Rightarrow x\geq3$

معکوس: $x=\sqrt{y-3} \Rightarrow x^2=y-3 \Rightarrow y=x^2+3 \Rightarrow f^{-1}(x)=x^2+3$

ب) $x^2-4x+3,\ x\in[2,+\infty) \Rightarrow$ راس سهمی $=\frac{-b}{2a}=\frac{4}{2}=2 \Rightarrow$ یک به یک است

$x=y^2-4y+3+1-1 \Rightarrow x=(y-2)^2-1 \Rightarrow x-1=(y-2)^2 \Rightarrow \sqrt{x-1}=y-2$

$y=\sqrt{x-1}+2 \Rightarrow f^{-1}(x)=\sqrt{x-1}+2$

3

(7)

$f(x) = x+\sqrt{x}$, $f^{-1}(x) = ax+b-\sqrt{x-c}$, $\sqrt{x} = t \Rightarrow f(x) = t^2+t$

$a+2b+4c = ?$ $\quad f^{-1}(x) \Rightarrow y = t^2+t+\frac{1}{4}-\frac{1}{4} \Rightarrow \left(t+\frac{1}{2}\right)^2-\frac{1}{4} = y \Rightarrow y = \left(\sqrt{x}+\frac{1}{2}\right)^2-\frac{1}{4}$

$D_f = [0,+\infty) = R_{f^{-1}} \;\Big|\Rightarrow x = \left(\sqrt{y}+\frac{1}{2}\right)^2-\frac{1}{4} \Rightarrow x+\frac{1}{4} = \left(\sqrt{y}+\frac{1}{2}\right)^2 \Rightarrow$

$\pm\sqrt{x+\frac{1}{4}} = \sqrt{y}+\frac{1}{2} \to \sqrt{x+\frac{1}{4}}-\frac{1}{2} = \sqrt{y} \Rightarrow y = x+\frac{1}{2}-\sqrt{x+\frac{1}{4}} \Rightarrow \begin{cases} a=1 \\ b=\frac{1}{2} \\ c=\frac{-1}{4} \end{cases}$

3

$\therefore a+2b+4c = 1+1-1 = 1$

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(8)

$y = \frac{x}{\sqrt{x^2-2}}$, $\frac{x_1}{\sqrt{x_1^2-2}} = \frac{x_2}{\sqrt{x_2^2-2}} \xrightarrow{*2} \frac{x_1^2}{x_1^2-2} = \frac{x_2^2}{x_2^2-2} \Rightarrow \cancel{x_1^2x_2^2}-2x_1^2 = \cancel{x_2^2x_1^2}-2x_2^2$

$\cancel{2}x_1^2 = \cancel{2}x_2^2 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

یک y نمی‌تواند دو x مختلف العلامت داشته باشد یعنی $x_1 = x_2$

معکوس: $f^{-1}(x) = \frac{y}{\sqrt{y^2-2}} \xrightarrow{*2} x^2 = \frac{y^2}{y^2-2} \Rightarrow x^2y^2-2x^2 = y^2 \Rightarrow x^2y^2-y^2 = 2x^2$

2

$y^2(x^2-1) = 2x^2 \Rightarrow y^2 = \frac{2x^2}{x^2-1} \Rightarrow y = \pm\sqrt{\frac{2x^2}{x^2-1}}$ $\;\underset{\text{هم‌علامت}}{y, x}$, $y = \frac{x\sqrt{2}}{\sqrt{x^2-1}}$

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(9)

$f^{-1}(x) = \frac{x}{1+|x|}$, $g(x) = \sqrt{x}-1$, $g^{-1}\left(-\frac{2}{5}\right)+f\left(-\frac{2}{5}\right)$

$g^{-1}(x) \Rightarrow \sqrt{x}-1 \Rightarrow y+1 = \sqrt{x} \Rightarrow x = (y+1)^2 \Rightarrow g^{-1}(x) = (x+1)^2 \Rightarrow g^{-1}\left(-\frac{2}{5}\right) = \left(-\frac{2}{5}+1\right)^2 = \frac{9}{25}$

$f^{-1}(x) \begin{cases} \frac{x}{1+x} \;;\; x \geq 0 \\ \frac{x}{1-x} \;;\; x<0 \end{cases} \Rightarrow y = \frac{x}{1-x} \Rightarrow f(x) \Rightarrow x = \frac{y}{1-y} \Rightarrow x-xy = y \Rightarrow x = y+xy$

$x = y(1+x) \Rightarrow y = \frac{x}{1+x} \Rightarrow f\left(-\frac{2}{5}\right) = \frac{-\frac{2}{5}}{1-\frac{2}{5}} = \frac{-\frac{2}{5}}{\frac{3}{5}} = \frac{-2}{3}$

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$\therefore g^{-1}\left(-\frac{2}{5}\right)+f\left(-\frac{2}{5}\right) = \frac{9}{25}-\frac{2}{3} = \frac{-23}{75}$

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(10)

$g(x) = f(x)+\sqrt{f(x)}$, $f^{-1}(x) = \sqrt[3]{x-1}$

$f^{-1}(x) = y = \sqrt[3]{x-1} \Rightarrow y^3 = x-1 \Rightarrow y^3-1 = x \Rightarrow f(x) = x^3-1$

$g(x) = x^3-1+\sqrt{x^3-1} \Rightarrow t = \sqrt{x^3-1} \Rightarrow g(x) = t^2+t \Rightarrow y = t^2+t \Rightarrow t^2+t-y = 0$

2

$t = \frac{-1\pm\sqrt{1+4y}}{2} \xrightarrow{\sqrt{x^3+1}\geq 0} t = \frac{-1+\sqrt{1+4y}}{2}$

$x^3+1 = t^2 \Rightarrow x^3 = t^2-1 \Rightarrow x = \sqrt[3]{t^2-1} \Rightarrow x = \sqrt[3]{\left(\frac{-1+\sqrt{1+4y}}{2}\right)^2-1} \Rightarrow g^{-1}(x) = \sqrt[3]{\left(\frac{-1+\sqrt{1+4x}}{2}\right)^2-1}$

$g^{-1}(12) = \sqrt[3]{\left(\frac{-1+\sqrt{49}}{2}\right)^2-1} = \sqrt[3]{8} = 2$