۸

۱- نگار عمرانی

(۱) $(m, 2) = (4, 2) \to m = 4$

الف $(4, n^2 - n) = (4, 2) \to n^2 - n - 2 = 0 \begin{cases} n = -1 \\ n = 2 \end{cases}$

(۲) ب

$(m, 4) = (3, 4) \to m = 3$

$(a, 1) = (-1, 1) \to a = -1$

$(9, n) = (9, 4) \to n = 4$

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(۲)

$D_f \to 4x - 1 \to R_f = [3, +\infty)$

$x \geq 1$

$f(x) = \cancel{x+1} \to R_f (1+a, \infty)$

$x < 1$

$1 + a < 2 \to \boxed{a \leq 1}$

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(۳)

$f(x): \begin{cases} 4x - 1 & x \geq 1 \\ ax + a - 1 & x \leq 0 \end{cases} \to R_f = [3, +\infty)$

$D_f = [a-1, +\infty)$

$a < 0$

$D_f = (-\infty, a-1]$ ✓

$a > 0$

(۱) $a - 1 < 3 \to 0 < a < 4$

(۲) $a > 0$

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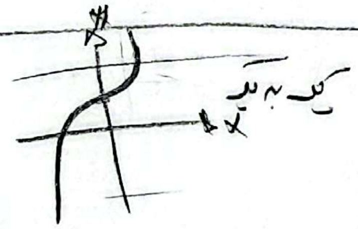
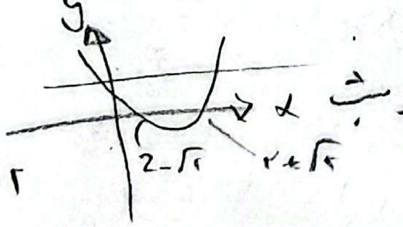
(۴)

الف $y = x^3 + 2$



(ب)



یک به یک نیست X

$y = (x-2)^2 - 2$

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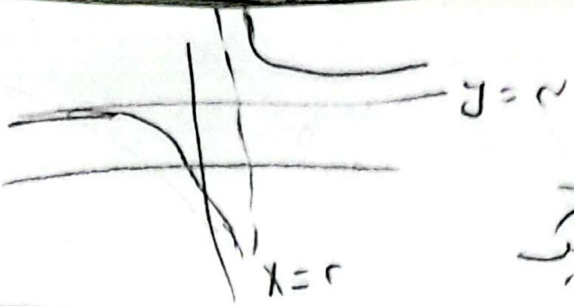
(۵)

ب $y = \frac{4 + 2x}{x + 2} = 2 \to$ تابع ثابت یک به یک نیست

$x = -2$

۵ الف

$$y=\frac{2x+1}{x-2}$$



یک به یک
[illegible]

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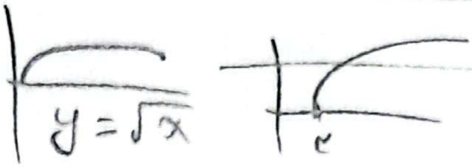
۶ الف

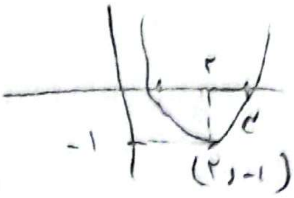
$$y=\sqrt{x-2}\longrightarrow$$

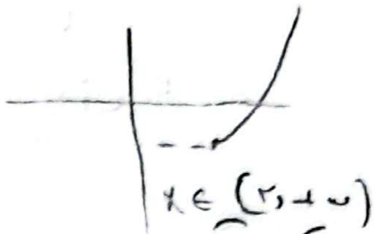


یک به یک

۶ ب

$$y=x^2-4x+3$$





یک به یک

۷

$$y=x+\sqrt{x}\longrightarrow(y-x)=\sqrt{x}\longrightarrow(y-x)^2=x$$

$$y^2-2xy+x^2-x=0\longrightarrow x^2-(2y+1)x+y^2=0$$

$$x=\frac{(2y+1)\pm\sqrt{(2y+1)^2-4y^2}}{2}$$

$$x=\begin{cases}\dfrac{2y+1+\sqrt{4y+1}}{2} & \longrightarrow f^{-1}(x)=x+\frac{1}{2}+\sqrt{x+\frac{1}{4}}\\[2ex] \dfrac{2y+1-\sqrt{4y+1}}{2} & \quad f^{-1}(x)=x+\frac{1}{2}-\sqrt{x+\frac{1}{4}}\end{cases}$$

$$ax+b-\sqrt{x-c}=x+\frac{1}{2}-\sqrt{x+\frac{1}{4}}$$

$a=1$

$b=\frac{1}{2}$

$c=-\frac{1}{4}$

$$a+2b+4c=1$$

$$1+1-1=1$$

$$\boxed{a+2b+4c=1}$$

8

$y=\frac{x}{\sqrt{x^2-2}} \Rightarrow D_f\,(-2,+2),\ R_f=(-\infty,0]\cup(1,+\infty)$

	0		1	
$2x$	$-$	$+$		$+$
$x-1$	$-$	$-$		$+$
	$+$	$-$		$+$

$y^2=\frac{x^2}{x^2-2} \to yx^2-2y^2=x^2 \to x^2(y^2-1)=2y$

$x^2=\frac{2y}{y-1} \to x=\pm\sqrt{\frac{2y^2}{y^2-1}}$

$f^{-1}(x)=\pm\sqrt{\frac{2x^2}{x^2-1}}\ ,\ D_{f^{-1}}(-\infty,0]\cup(1,+\infty)$

$R_{f^{-1}}(-2,+2)$

$y=\frac{x_1}{\sqrt{x_1^2-2}}$

$y=\frac{x_2}{\sqrt{x_2^2-2}} \to \frac{x_1}{\sqrt{x_1^2-2}}=\frac{x_2}{\sqrt{x_2^2-2}} \quad \frac{x_1^2}{x_1^2-2}=\frac{x_2^2}{x_2^2-2} \to \cancel{1}\ \ x_1=x_2$

9

$g(x)=\sqrt{x}-1 \to y=\sqrt{x}-1 \to \sqrt{x}=+1+y$

$x=(1+y)^2 \to g^{-1}(x)=(1+x)^2$

$-\frac{2}{5}=\sqrt{x}-1 \to \frac{3}{5}=\sqrt{x} \Rightarrow \boxed{x=\frac{9}{25}}$

$\to g^{-1}(-\frac{2}{5})=\frac{9}{25}$

$\frac{x}{1+|x|}=-\frac{2}{5} \to 5x=-2-2|x|$

$x>0 \to 7x=-2 \to \boxed{x=-\frac{2}{7}}$

$x<0 \quad 3x=-2 \to \boxed{x=-\frac{2}{3}}$

$f(-\frac{2}{5})^{-1}= \cancel{-\frac{2}{7}} \quad -\frac{2}{3}$

$\frac{-\frac{2}{7}}{1+\frac{2}{7}}=\frac{-\frac{2}{7}}{\frac{9}{7}}=-\frac{2}{9}$

$\boxed{f(-\frac{2}{5})=-\frac{2}{3}}$

$\frac{-\frac{2}{3}}{1-\frac{2}{3}}=\frac{-\frac{2}{3}}{\frac{1}{3}}=2$

$g^{-1}(\frac{2}{5})=\frac{9}{25}$

$g(x)$

$g(x)\quad f(-\frac{2}{5})+g^{-1}(-\frac{2}{5})=-\frac{2}{3}+\frac{9}{25}=-\frac{23}{75}$

جواب $-\frac{23}{75}$

(۱)

$f^{-1}(x) = \sqrt[۴]{x-1} \to y = \sqrt{x^۴ - 1} \to f(x) = x^۴ + 1$

$g(x) = x^۴ + 1 + \sqrt{x^۴ + 1} = ۱۲ \to x = ۲$

$g(۲) = ۱۲ \to g^{-1}(۱۲) = ۲$