

الف) $y = \sqrt{x-1}$

$$\left. \begin{aligned} y &= \sqrt{x_1-1} \\ y &= \sqrt{x_2-1} \end{aligned} \right\} \sqrt{x_1-1} = \sqrt{x_2-1} \Rightarrow x_1-1 = x_2-1 \Rightarrow x_1 = x_2$$

ب) $y = x^2 - 2x + 1$ $x \in [1, \infty)$

$\frac{f}{y} = 1$

$y = (x-1)^2 - 1$

$x = (y+1)^2 - 1 \Rightarrow x-1 = (y+1)^2$

$\sqrt{x-1} = y+1$

$f'(x) = y = \sqrt{x-1} + 1$

$f(x) = x + \sqrt{x} \Rightarrow R = (0, \infty) \rightarrow$

$f'(x) = a + b\sqrt{x} - c = 1 = (0, \infty)$

$f'(x) \Rightarrow x = y + \sqrt{y} \xrightarrow{\text{تفاضل}} x = y + \sqrt{y} + \frac{1}{2\sqrt{y}} - \frac{1}{2\sqrt{y}} = x + \frac{1}{2\sqrt{y}} = \left(\sqrt{y} + \frac{1}{2\sqrt{y}}\right)^2 = \sqrt{y} + \frac{1}{\sqrt{y}}$

$\Rightarrow \sqrt{y} = \frac{1}{2\sqrt{y}} + \sqrt{x + \frac{1}{2\sqrt{y}}} \Rightarrow y = \frac{1}{4} + x + \frac{1}{4} - \frac{1}{2\sqrt{y}} \sqrt{x + \frac{1}{2\sqrt{y}}} \Rightarrow y = x - \sqrt{x + \frac{1}{2\sqrt{y}}} + \frac{1}{2\sqrt{y}} = ax + b\sqrt{x-c}$

$a + 2b + 4c = 1 + 1 - 1 = 1$ $a=1, b=\frac{1}{2}, c=-\frac{1}{4}$

$y = \frac{x}{\sqrt{x^2-1}} \Rightarrow y = y \Rightarrow \frac{x_1}{\sqrt{x_1^2-1}} = \frac{x_2}{\sqrt{x_2^2-1}} \xrightarrow{\text{كسر}} \frac{x_1^2}{x_1^2-1} = \frac{x_2^2}{x_2^2-1} \Rightarrow \frac{x_1^2 + 1 - 1}{x_1^2-1} = \frac{x_2^2 + 1 - 1}{x_2^2-1}$

$\Rightarrow 1 + \frac{1}{x_1^2-1} = 1 + \frac{1}{x_2^2-1} \Rightarrow \frac{1}{x_1^2-1} = \frac{1}{x_2^2-1} \Rightarrow x_1^2-1 = x_2^2-1 \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm x_2$

ملاحظه: $x = \frac{y}{\sqrt{y^2-1}} \Rightarrow x^2 = \frac{y^2}{y^2-1} \Rightarrow x^2 y^2 - 1 = y^2 \Rightarrow y^2 = \frac{1}{x^2-1} \Rightarrow y = \frac{\sqrt{1-x^2}}{x^2-1}$

$f'(x) = \frac{x}{1+x^2}, g(x) = \sqrt{x} - 1 \Rightarrow f(x) = \begin{cases} x > 0 \\ x < 0 \end{cases} \Rightarrow f(-\frac{1}{2}) = -\frac{1}{2}$

$f(-\frac{1}{2}) + g'(-\frac{1}{2})$ $g(x) = y = x+1 \Rightarrow y = (x+1)^2 \Rightarrow g'(x) = 2(x+1) \Rightarrow g'(-\frac{1}{2}) = \frac{1}{2}$

$\left(-\frac{1}{2} + \frac{1}{2} = \frac{-50+17}{\sqrt{5}} = -\frac{33}{\sqrt{5}} \right)$

$f^{-1}(x) = \sqrt{x-1} \Rightarrow x = \sqrt{y-1} \Rightarrow y-1 = x^2 \Rightarrow y = x^2 + 1$

$g(x) = f(x) + \sqrt{f(x)} \xrightarrow{x > -1} g(x) = x^2 + 1 + \sqrt{x^2 + 1} = 12 \Rightarrow x^2 + \sqrt{x^2 + 1} = 11 \Rightarrow \sqrt{x^2 + 1} = 11 - x^2 \Rightarrow x^2 + 1 = (11 - x^2)^2 \Rightarrow x^2 + 1 = 121 - 22x^2 + x^4 \Rightarrow x^4 - 23x^2 + 120 = 0$

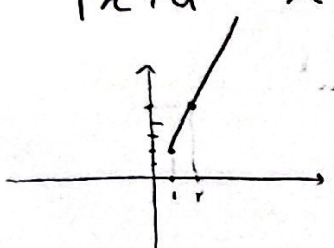
$g^{-1}(12) = a \Rightarrow g(a) = 12 \Rightarrow g'(a) = 2$

الف) $F = \{(3, 2)(n, 5)(3, n^2 - n)(m, 2)(-1, 4)\}$

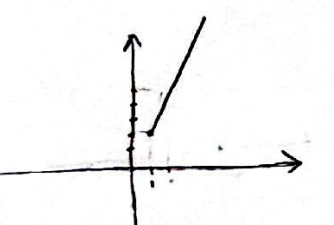
$n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow (n-2)(n+1) = 0$
 $\begin{matrix} \text{①} & \text{②} \end{matrix} \rightarrow \begin{matrix} -1 \\ 2 \end{matrix} \rightarrow \begin{matrix} (-1, 5) \\ (-1, 4) \end{matrix} \Rightarrow n=2$

ب) $F = \{(-1, 1)(1, 2)(2, 3)(a, m-1)(a+1, n)(m, 2)\} \rightarrow n=2$
 $m=2 \quad a+1=1$
 $a=-1 \quad n=2$

$f(x) = \begin{cases} 2x-1 & x \geq 1 \\ x+a & x < 1 \end{cases} \rightarrow (-\infty, 2) \rightarrow x+a < 2 \Rightarrow a > 1$



$f(x) = \begin{cases} 2x-1 & x \geq 1 \\ ax+a-1 & x \leq 0 \end{cases} \rightarrow (-\infty, 2) \rightarrow ax+a-1 < 2 \rightarrow a-1 < 2 \rightarrow a < 3$



الف) $y = x^3 + 2$
 $y_1 = x_1^3 + 2$
 $y_2 = x_2^3 + 2$
 $x_2^3 + 2 = x_1^3 + 2 \rightarrow x_2 = x_1$
 $x = y^3 + 2 \rightarrow y^3 = x - 2 \rightarrow y = \sqrt[3]{x-2}$

ب) $y = x^3 - 2x + 2 + 2 - 2$
 $y = (x_1 - 2)^3 - 2$
 $y = (x_2 - 2)^3 - 2$
 $(x_1 - 2)^3 - 2 = (x_2 - 2)^3 - 2 \rightarrow x_1 = x_2$
 تابع یک به یک نیست

الف) $y = \frac{2x+1}{x-2}$ تابع همگراست یک به یک است
 $x = \frac{-2y+1}{y-2} \rightarrow y = \frac{2x+1}{x-2}$

ب) $y = \frac{2+2x}{x+2}$ تابع ثابت ۲ است، یک به یک نیست
 $\frac{2(x+2)}{x+2} = 2$