

الف) $f = \{ (2, 2), (n, 0), (2, n^2 - n), (m, 2), (-1, 4) \}$

$m = 2$

$n^2 - n = 2 \rightarrow n^2 - n - 2 = 0 \rightarrow (n - 2)(n + 1) = 0 \rightarrow \begin{cases} n = 2 \\ n = -1 \end{cases}$
که غیر قابل قبول

ب) $f = \{ (-1, 1), (1, 2), (2, 3), (a, m - 1), (a + 2, n), (n, 2) \}$

$a = -1$
 $m = 2$
 $n = 2$

گراف شرا باشد
تابع یک به یک نیست

دستور مقدار a
 $a \in (-\infty, 2]$

$f(n) = \begin{cases} 2n - 1 & n \geq 1 \\ n + a & n < 1 \end{cases}$

$f(n) = \begin{cases} 2n - 1 & n \geq 1 \\ a + n - 1 & n \leq 0 \end{cases}$

تابع یک به یک نیست
 $a - 1 < 2 \rightarrow a < 3$

الف) $y = x^3 + 2$

$y = x_1^3 + 2$
 $y = x_2^3 + 2$
 $\sqrt[3]{y - 2} = \sqrt[3]{x_1^3} = x_1$
 $\sqrt[3]{y - 2} = \sqrt[3]{x_2^3} = x_2$

$x_1 = x_2$

$x = y^3 + 2 \rightarrow y^3 = x - 2 \rightarrow y = \sqrt[3]{x - 2}$

ب) $y = x^2 - 4x + 2 \rightarrow$ درجه دار سه یک به یک نیست

الف) $y = \frac{2n + 1}{n - 2}$

تابع فرد گرافیک یک به یک نیست

$n = \frac{2y + 1}{y - 2} \rightarrow ny - 2n = 2y + 1$

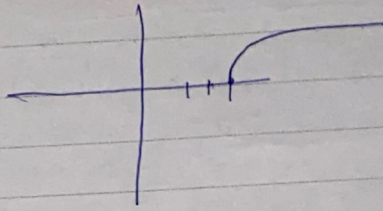
$(n - 2)y = 2n + 1$

$y = \frac{2n + 1}{n - 2}$

ب) $y = \frac{2n + 4}{n + 2} = \frac{2(n + 2)}{n + 2} = 2 \rightarrow$ تابع ثابت است پس یک به یک نیست

سوال 4

الف) $y = \sqrt{x-r}$
 $(x = \sqrt{y-r})^2$



$$x^2 = y - r \rightarrow y = x^2 + r$$

ب) $y = x^2 + rx + r \quad x \in [r, +\infty)$ تعیین است

$$\frac{-b}{2a} = \frac{r}{2} = r$$

$$y^2 + y + r(x-x) = 0 \rightarrow y = \frac{-1 \pm \sqrt{1 - 4(-r)(x-x)}}{2} = \frac{-1 \pm \sqrt{4rx}}{2} = -\frac{1}{2} \pm \sqrt{rx}$$

از آنجا که $x \geq r \rightarrow -\frac{1}{2} \pm \sqrt{rx} \geq r$

$$y = r + \sqrt{rx+1}$$

سوال 5

$f(x) = x\sqrt{x}$

$$f'(x) = x = y + \sqrt{y + \frac{1}{x}} - \frac{1}{x} \rightarrow (\sqrt{y + \frac{1}{x}})^2 - \frac{1}{x} \rightarrow x + \frac{1}{x} = (y + \frac{1}{x})^2$$

$$\sqrt{y + \frac{1}{x}} = \sqrt{x + \frac{1}{x}} \rightarrow \sqrt{y} = \sqrt{x + \frac{1}{x}} - \frac{1}{\sqrt{x}} \rightarrow y = x + \frac{1}{x} + \frac{1}{x} - \sqrt{x + \frac{1}{x}} = x + \frac{2}{x} - \sqrt{x + \frac{1}{x}}$$

$$x - \sqrt{x + \frac{1}{x}} + \frac{1}{x} \rightarrow \begin{cases} a=1 \\ b=\frac{1}{x} \\ c=-\frac{1}{x} \end{cases}$$

$$a + b + c = 1 + 1 - 1 = 1$$

$y = \frac{x}{\sqrt{x^2 - r}}$ تعیین است

سوال 6

$$\left. \begin{aligned} y &= \frac{x}{\sqrt{x^2 - r}} \\ y &= \frac{x^2}{\sqrt{x^2 - r}} \end{aligned} \right\} \frac{x}{\sqrt{x^2 - r}} = \frac{x^2}{\sqrt{x^2 - r}}$$

اولی را در دومی ضرب می‌کنیم

$$x = \frac{y}{\sqrt{y^2 - r}} \rightarrow x^2 = \frac{y^2}{y^2 - r} \rightarrow x^2 y^2 - r x^2 = y^2 \rightarrow (x^2 - 1) y^2 = r x^2$$

$$\frac{x^2}{x^2 - 1} = \frac{y^2}{y^2 - r}$$

$$x^2 y^2 - x^2 = y^2 - r y^2 \rightarrow x^2 y^2 - y^2 = x^2 - r y^2$$

$$(x^2 - 1) y^2 = r x^2$$

$$x = y \leftarrow \sqrt{x^2} = \sqrt{y^2}$$

$$y^2 = \frac{r x^2}{x^2 - 1} \rightarrow y = \pm \frac{\sqrt{r} x}{\sqrt{x^2 - 1}} \rightarrow y = \frac{\sqrt{r} x}{\sqrt{x^2 - 1}}$$

$\sqrt{x^2 - 1} \geq 0$ از آنجا که $x \geq 1$

NOTE

$$f(n) = \frac{n}{1+|n|} \quad g(n) = \sqrt{n} - 1 \quad f\left(-\frac{r}{a}\right) + g^{-1}\left(-\frac{r}{a}\right) = -\frac{r}{r+a} \quad (9)$$

$$n = \frac{y}{1+|y|} \rightarrow n+|y|n = y \rightarrow y - |y|x = n \xrightarrow{y \geq 0} \frac{-yr}{\sqrt{y}} \rightarrow y - yx = n \rightarrow y(1-x) = n$$

$$y < 0 \rightarrow y + yx = n \rightarrow n = y(n+1) \rightarrow y = \frac{n}{n+1}$$

$$f(n) = \begin{cases} \frac{n}{1+n} & n \geq 0 \\ \frac{n}{n+1} & n < 0 \end{cases} \rightarrow f\left(-\frac{r}{a}\right) = \frac{-r}{a} = \frac{-r}{a} = -\frac{r}{a}$$

$$n = \sqrt{y} - 1 \rightarrow \sqrt{y} = n+1 \rightarrow y = (n+1)^2 \rightarrow g(n) = (n+1)^2 \rightarrow g^{-1}\left(-\frac{r}{a}\right) = \left(\frac{r}{a}\right)^2 = \frac{r^2}{a^2}$$

$$f(n) = \sqrt{n-1} \quad g(n) = f(n) + \sqrt{f(n)}$$

$$g^{-1}(r) = ? \quad (n = \sqrt[r]{y-1})^r \rightarrow n^r = y-1 \rightarrow y = n^r + 1$$

$$g(n) = n^r + 1 + \sqrt[n^r+1]{n^r+1} \quad g^{-1}(n) \rightarrow n = y^r + 1 + \sqrt[y^r+1]{y^r+1} \quad \sqrt[y^r+1]{y^r+1} = t$$

$$n = t^r + 1 \rightarrow t^r + 1 - n = 0$$

$$t = \frac{-1 \pm \sqrt{1+n}}{r} \quad t \geq 0 \rightarrow t = \frac{\sqrt{1+n} - 1}{r}$$

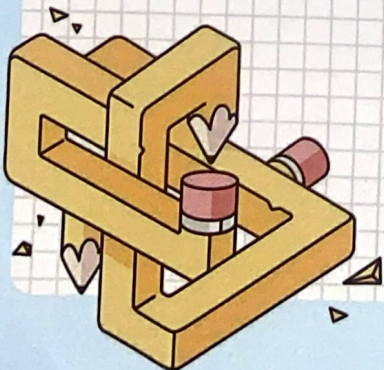
$$\sqrt[y^r+1]{y^r+1} = \left(\frac{\sqrt{1+n} - 1}{r} \right)^r \rightarrow y^r + 1 = \frac{r(n+1) - r\sqrt{1+n}}{r}$$

$$y^r + 1 = \frac{r(n - \sqrt{1+n}) + 1}{r} \rightarrow y^r = \frac{r(n - \sqrt{1+n}) - 1}{r}$$

$$y = \sqrt[r]{\frac{r(n - \sqrt{1+n}) - 1}{r}} \rightarrow n = r \rightarrow f^{-1}(r) = \sqrt[r]{\frac{r(r - \sqrt{1+r}) - 1}{r}} =$$

$$\sqrt[r]{\frac{r(r - r - 1)}{r}} =$$

$$\sqrt[r]{\frac{r(-1)}{r}} = \sqrt[r]{-1} = -1$$



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