

(V)

$$f(u) = y \Rightarrow u + \sqrt{u} \xrightarrow{\text{square}} y + \sqrt{y} = u \Rightarrow u + \frac{1}{2} = \sqrt{y + \frac{1}{2}} + \frac{1}{2} \rightarrow \frac{1}{2} + u = \left(\sqrt{y + \frac{1}{2}} + \frac{1}{2}\right)^2$$

$$\sqrt{u + \frac{1}{2}} = \sqrt{y + \frac{1}{2}} + \frac{1}{2} \rightarrow \sqrt{y} = \sqrt{u + \frac{1}{2}} - \frac{1}{2}$$

$$y = \left(\sqrt{u + \frac{1}{2}} - \frac{1}{2}\right)^2 \rightarrow y = \frac{1}{4} + u + \frac{1}{4} - \sqrt{u + \frac{1}{2}} \rightarrow y = \frac{1}{2} + u - \sqrt{u + \frac{1}{2}}$$

$$b = \frac{1}{2} \quad a = 1 \quad c = -\frac{1}{2}$$

$$a + b + c = 1 + 1 - 1 = 1$$

(1)

$$y = \frac{u}{\sqrt{u^2 - r}} \rightarrow f(u_1) = f(u_r) \Rightarrow \frac{u_1}{\sqrt{u_1^2 - r}} = \frac{u_r}{\sqrt{u_r^2 - r}}$$

$$u_1^2 (u_r^2 - r) = u_r^2 (u_1^2 - r) \Rightarrow u_1 = \pm u_r \rightarrow u_1 = u_r$$

if $u_1 = u_r$ then y is not unique $\Rightarrow$ we need $u_1 \neq u_r$

$$\xrightarrow{\text{square}} x = \frac{y}{\sqrt{y^2 - r}} \Rightarrow x^2 (y^2 - r) = y^2 \rightarrow x^2 y^2 - r x^2 = y^2$$

$$y^2 (x^2 - 1) = r x^2$$

$$y = \pm \sqrt{\frac{r x^2}{x^2 - 1}} \Rightarrow \frac{\sqrt{r} x}{\sqrt{x^2 - 1}}$$

(9)

$$f(u) = \frac{u}{1+|u|} \quad g(u) = \sqrt{u} - 1 \quad f\left(-\frac{r}{\omega}\right) + g\left(-\frac{r}{\omega}\right)$$

$$-\frac{r}{\omega} = \frac{u}{1+|u|} \xrightarrow{u < 0} \frac{u}{1-u} \Rightarrow \frac{-r}{\omega} = \frac{u}{1-u} \Rightarrow -r(1-u) = \omega u \Rightarrow -r + ru = \omega u \Rightarrow ru - \omega u = r \Rightarrow u(r - \omega) = r \Rightarrow u = \frac{r}{r - \omega}$$

$$\sqrt{u} - 1 = -\frac{r}{\omega} \rightarrow \sqrt{u} = \frac{r}{\omega} - 1 \rightarrow u = \left(\frac{r}{\omega} - 1\right)^2$$

$$\frac{-r \times r}{\omega \times \omega} + \frac{r \times r}{r \times r} = \frac{-r^2}{\omega^2} + \frac{r^2}{r^2} = \frac{-r^2}{\omega^2} + 1 = \frac{-r^2 + \omega^2}{\omega^2}$$

(10)

$$f(u) = \sqrt{u-1} \quad g(u) = f(u) + \sqrt{f(u)}$$

$$y = \sqrt{u-1} \quad y(u) = u^r + 1 + \sqrt{u^r + 1}$$

$$y^r + 1 = u \quad 1^r = u^r + 1 + \sqrt{u^r + 1}$$

$$\boxed{u=r} \rightarrow a + \sqrt{a}$$

$$f(u) = u^r + 1$$