

$$A \times B - B^2$$

$$\{(1,2)(1,3)(2,2)(2,3)(3,2)(3,3)\} - \{(2,2)(3,3)(3,2)(2,3)\} \rightarrow$$

$$\rightarrow \{(1,2)(1,3)\}$$

1

$$\text{الف) } \{(3, m+2)(2,1)(-3, m)(-2, m)(3, m+2)(m, 3)\} \rightarrow m^2 = m+2$$

$$m^2 - m - 2 \rightarrow m(m-1) = 2 \rightarrow m=2 \checkmark$$

م و توان 2 باشد و برای m عددی 3 غلط

ب. -

2

$$\text{ب) } \{(3, m+2)(2,1)(m, 3)(3, m+2)(-2, m)(-1, 3)\} \rightarrow m^2 = m+2$$

$$m^2 - m - 2 \rightarrow m(m-1) = 2 \rightarrow m=2 \checkmark$$

م و توان 2 باشد و برای m عددی 3 غلط

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$$f(x) = \begin{cases} x^2 - 1 & ; x \leq 1 \\ ax + b & ; 1 < x \leq 2 \\ x^3 & ; x > 2 \end{cases} \rightarrow \begin{cases} x^2 - 1 = ax + b \\ ax + b = x^3 \end{cases} \rightarrow \begin{cases} 1 - 1 = a + b & -(a + b = 0) \\ 2a + b = 8 & 2a + b = 8 \end{cases}$$

$$\begin{aligned} a &= 8 \\ 8 + b &= 0 \\ b &= -8 \end{aligned}$$

$$\rightarrow ax + b \rightarrow 8x - 8 = -4f$$

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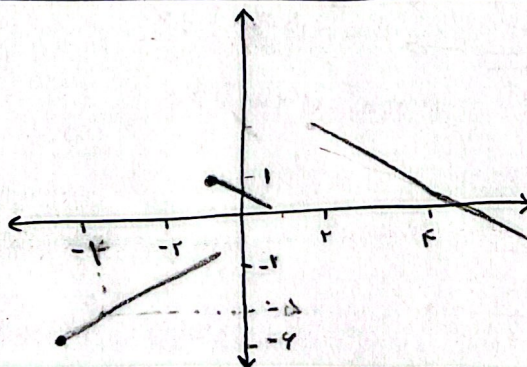
$$f(x) = \begin{cases} 2x - 3 & ; x \geq k \\ f - 3x & ; x \leq k \end{cases} \rightarrow \begin{cases} 2k - 3 = f - 3k \\ 2k + 3k = f + 3 \end{cases} \rightarrow 5k = f + 3 \rightarrow k = \frac{f+3}{5}$$

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$$f(x) = \begin{cases} -x + 2 & ; x > 1 \\ -x & ; -1 \leq x < 1 \\ x - 1 & ; x < -1 \end{cases}$$

$$\text{دامنه} \rightarrow \mathbb{R} / (-\infty, +\infty)$$

$$\text{بردار} \rightarrow (-\infty, 1]$$



$$2y^3 + x^2 + 1 = 0 \rightarrow 2y^3 = -x^2 - 1 \rightarrow y^3 = -\frac{x^2+1}{2} \rightarrow y = \sqrt[3]{-\frac{x^2+1}{2}}$$

$$\rightarrow y_1 = y_2 \rightarrow \text{تابع است} \checkmark$$

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$$\text{ب) } x^2 + 4y^2 - 2x - 12y + 10 = 0 \rightarrow 4y^2 = -x^2 + 2x + 12y - 10 \text{ و } 4y^2 = -x^2 + 2x + 12y - 10$$

$$4y^2 = 4y^2 \rightarrow y_1 = \pm y_2 \rightarrow \text{تابع نیست}$$

$$\text{الف) } x \sin y = y \sin x \xrightarrow{\text{توانیم در دو طرف تقسیم کنیم}} \frac{x}{\sin x} = \frac{y}{\sin y} \rightarrow \frac{1}{\sin 1} = \frac{1}{\sin y} \rightarrow \text{تابع نیست } x$$

در دو طرف تقسیم کردیم

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$$\text{ب) } \sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 2 \xrightarrow{\text{توانیم در دو طرف توان دوم بگیریم}} \left(t + \frac{1}{t} = 2\right) \times t \rightarrow t^2 + 1 = 2t \rightarrow t^2 + 1 - 2t = 0 \rightarrow (t-1)^2 = 0 \rightarrow t = 1$$

$$\rightarrow \sqrt{\frac{x}{y}} = 1 \rightarrow \frac{x}{y} = 1 \rightarrow \boxed{x=y} \rightarrow \text{تابع است} \checkmark$$

$$\text{الف) } \sqrt{\frac{x-1}{x-3}} + \sqrt{\frac{3-x}{x}} = \dots \rightarrow \frac{x-1}{x-3} \geq 0 \rightarrow \left| \frac{1}{+0} - \frac{3}{-} \right| \rightarrow (-\infty, 1] \cup (3, +\infty)$$

$$\frac{3-x}{x} \geq 0 \rightarrow \left| \frac{3}{+} - \frac{1}{-} \right| \rightarrow (0, 3]$$

اینجا: $[0, 3]$

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$$\text{ب) } \sqrt{x} + \sqrt{y-1} = \sqrt{3} \rightarrow x \geq 0 \rightarrow \sqrt{x} \leq \sqrt{3} \rightarrow x \leq 3 \rightarrow y-1 = (\sqrt{3}-\sqrt{x})^2 \geq 0$$

$$\rightarrow \sqrt{3}-\sqrt{x} \geq 0 \rightarrow \sqrt{x} \leq \sqrt{3} \rightarrow x \leq 3 \quad y = 1 + (\sqrt{3}-\sqrt{x})^2$$

$$\rightarrow \text{تابع است} \rightarrow \{(x, y) \mid 0 \leq x \leq 3, y = 1 + (\sqrt{3}-\sqrt{x})^2\}$$

$$\text{الف) } y = \frac{\sqrt{x^2 - 7x + 4}}{\sqrt{x - 10x + 14}} \rightarrow \frac{\sqrt{(x-4)(x-1)}}{\sqrt{-9x+14}} \rightarrow (x-4)(x-1) \geq 0 \rightarrow x \leq 1 \text{ یا } x \geq 4$$

$$\sqrt{-9x+14} \neq 0 \rightarrow -9x+14 \neq 0 \rightarrow x \neq \frac{14}{9}$$

$$-9x+14 \geq 0 \rightarrow 14 \geq 9x \rightarrow x \leq \frac{14}{9} \rightarrow (-\infty, \frac{14}{9}]$$

$$\text{ب) } y = \frac{\sqrt{x^2 - 7x + 4}}{\sqrt{x^2 - 10x + 14}} \rightarrow \frac{x^2 - 7x + 4}{x^2 - 10x + 14} \geq 0$$

$$\rightarrow (x-4)(x-1) \geq 0 \rightarrow x = 4 \text{ یا } x = 1$$

$$\rightarrow (x-1)(x-2) \geq 0 \rightarrow x = 1 \text{ یا } x = 2$$

$$\text{الف) } \sqrt{\frac{2x^2 - 5x + 3}{3x^2 - 7x + 4}} \rightarrow 2x^2 - 5x + 3 \geq 0 \rightarrow \Delta = (-5)^2 - 4 \times 2 \times 3 = 1 \rightarrow (-\infty, 1] \cup [\frac{3}{2}, +\infty)$$

$$\frac{5 \pm 1}{4} = \frac{6}{4} = \frac{3}{2} \text{ یا } \frac{4}{4} = 1$$

$$3x^2 - 7x + 4 \geq 0 \rightarrow \Delta = 49 - 4 \times 3 \times 4 = 1 \rightarrow (-\infty, 1] \cup [\frac{4}{3}, +\infty)$$

$$\frac{7 \pm 1}{6} = \frac{8}{6} = \frac{4}{3} \text{ یا } \frac{6}{6} = 1$$

$$\text{ب) } \sqrt{\frac{2x^2 - 5x + 3}{3x^2 - 7x + 4}} \rightarrow \frac{(2x-3)(x-1)}{(3x-4)(x-1)} \geq 0 \rightarrow x = \frac{3}{2} \text{ یا } x = \frac{4}{3}$$

$$\rightarrow (-\infty, 1] \cup (1, \frac{4}{3}) \cup [\frac{3}{2}, +\infty)$$