۱-

$A = \{1, 2, 4\}$

$B = \{2, 4\}$

$A \times B = \{(1,2)(1,4), \cancel{(2,2)}\cancel{(2,4)}\cancel{(4,2)}\cancel{(4,4)}\}$

$B^2 = \{(2,2)(2,4)(4,2)(4,4)\}$

$A \times B - B^2 = \{(1,2),(1,4)\}$

۲-

الف- $(3, m^2) = (3, m+2) \rightarrow m^2 - m - 2 = 0$

$(m+1)(m-2) = 0 \rightarrow \begin{cases} m=-1 \ ✓ \\ m=2 \ ✗ \end{cases}$

فقط $m=-1$ قابل قبول

ب- $(3, m^2) = (2, m+2) \rightarrow m^2 - m - 2 = 0$

$(m-2)(m+1) \rightarrow \begin{cases} m=-1 \ ✗ \\ m=2 \ ✗ \end{cases}$

هیچ مقدار

۳-

$$f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ ax+b & 1 \leq x < 2 \\ x^2 & x \geq 2 \end{cases}$$

$f(1) = 0 \rightarrow a = -b$

$f(1) = a+b$

$f(2) = 4 \rightarrow 2a+b = 4$

$f(2) = 2a+b$

$\rightarrow a = +8$, $b = -8$

$ab = -64$

۴-

$$f(x) = \begin{cases} 2x-3 & x \geq k \\ 4-3x & x \leq k \end{cases}$$

$f(k) = 2k - 3 = 4 - 3k$

$5k = 7 \rightarrow k = \frac{7}{5}$

۵-

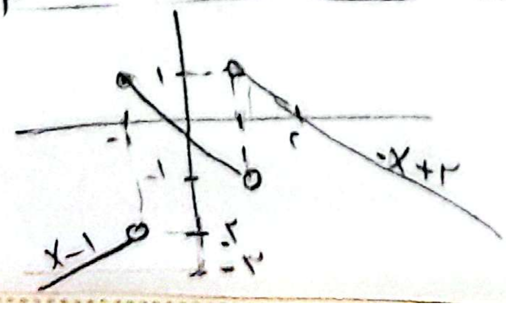
الف) $2y^3 + x^2 + 1 = 0 \rightarrow y = \sqrt[3]{\frac{-1-x^2}{2}}$ تابع است

ب) $x^2 + 4y^2 - 2x - 12y + 1 = 0 \rightarrow (x-1)^2 + (2y-3)^2 = 0$

فقط در یک نقطه تابع است $x = 1$, $y = \frac{3}{2}$

۶

$$f(x) = \begin{cases} -x+2 & x > 1 \\ -x & -1 \leq x < 1 \\ x-1 & x < -1 \end{cases}$$



۷-

الف) $x\sin y = y\sin x \to \frac{\sin y}{y} = \frac{\sin x}{x}$

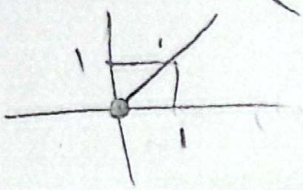
$x=\frac{\pi}{4} \to \frac{\sin y}{y} = \frac{\frac{\sqrt{2}}{2}}{\frac{\pi}{4}} \to y=\frac{\pi}{4}$

تابع نباشد - تابع هست
به ازای $x=0$

ب) $\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}=2$

$\left(\sqrt{\frac{x}{y}}+\sqrt{\frac{y}{x}}\right)^2=4 \to \frac{x}{y}+\frac{y}{x}+2=4 \to y^2+x^2-2xy=0$

$(x-y)^2=0$



$x=y$ به ازای $x>0$، $x\neq 0$، $y\neq 0$ ← تابع است.

$x>0$

۸-

$y=\sqrt{\frac{x-1}{x-3}}+\sqrt{\frac{2-x}{x}}$ $\qquad \frac{x-1}{x-3}\geqslant 0$

x		1		3	
$x-1$	$-$	0	$+$		$+$
$x-3$	$-$		$-$	0	$+$
$P_1(x)$	$+$		$-$		$+$

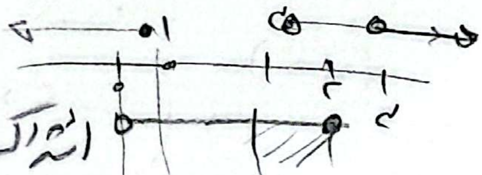
الف

$\frac{2-x}{x}\geqslant 0 \longrightarrow$

x		0		2	
$2-x$	$+$		$+$	0	$-$
x	$-$	0	$+$		$+$
$P_2(x)$	$-$		$+$		$-$

$P_1(x) \leadsto (-\infty,1]\cup(3,+\infty)$

$P_2(x) \leadsto (0,2]$



اشتراک $\leadsto$ $(0,1]$

ب) $\sqrt{x}+\sqrt{y-1}=\sqrt{3} \to \sqrt{y-1}=\sqrt{3}-\sqrt{x}$

$y=(\sqrt{3}-\sqrt{x})^2+1 \to \sqrt{x} \to x\geqslant 0 \quad [0,+\infty)$

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ب) ۹

$$y=\sqrt{\frac{x^2-7x+6}{x^2-10x+16}}$$

① $x^2-10x+16\neq 0$

② $\dfrac{x^2-7x+6}{x^2-10x+16}\geq 0$

$(x-2)(x-8)>0$

$(x-1)(x-6)\geq 0$

ب. مهرانی سمار

x		1		2		6		8	
x^2-7x+6	+	0	−		−	0	+		+
$x^2-10x+16$	+		+	0	−		−	0	+
	+	0	−	0	+	0	−	0	+

$D_f=(+\infty,1]\cup(2,6]\cup(8,+\infty)$

الف) ۹

$$\frac{\sqrt{x^2-7x+6}}{\sqrt{x^2-10x+16}}\rightsquigarrow$$

① $x^2-7x+6\geq 0$

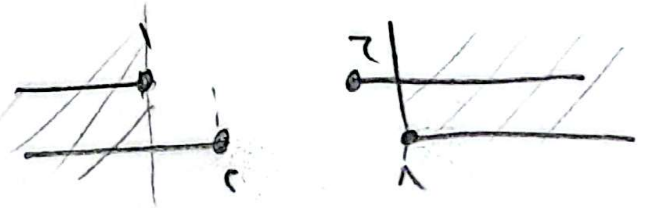
② $x^2-10x+16\geq 0$

x		1		6	
x^2-7x+6	+	0	−	0	+

x		2		8	
$x^2-10x+16$	+		−		+

$D_f=(-\infty,1]\cup[6,+\infty)$ $\quad D_g=(+\infty,2]\cup[8,+\infty)$

اشتراک $D_f\cap D_g\rightarrow$



$D_f\cap D_g=(+\infty,1]\cup[8,+\infty)$

۱۰

الف) $\dfrac{\sqrt{2x^2-5x+3}}{\sqrt{3x^2-7x+4}}$ $\quad$ ① $2x^2-5x+3 \geq 0$ $\quad$ ② $3x^2-7x+4 > 0$

$2x^2-5x+3 = (x-1)(2x-3) \to x=+1,\ x=+\frac{3}{2}$

$3x^2-7x+4 = (x-1)(3x-4) = 0 \quad x=1,\ x=\frac{4}{3}$

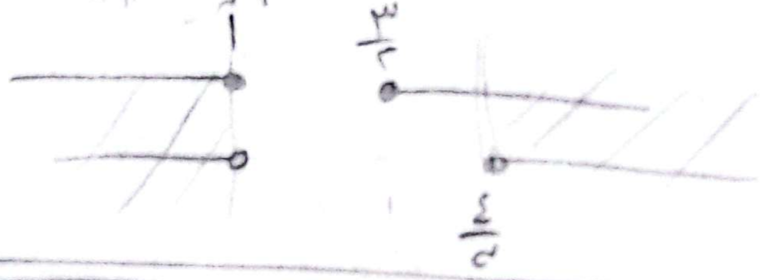
x		$+1$		$\frac{3}{2}$	
$2x^2-5x+3$	$+$	0	$-$	0	$+$

$D_f = (-\infty,+1] \cup [\frac{3}{2},+\infty)$

x		1		$\frac{4}{3}$	
$3x^2-7x+4$	$+$	0	$-$	0	$+$

$D_g = (-\infty,1) \cup (\frac{4}{3},+\infty)$

$D_f \cap D_g \Rightarrow (-\infty,1) \cup (\frac{4}{3},+\infty)$



۱۰ ب) $y=\sqrt{\dfrac{2x^2-5x+3}{3x^2-7x+4}} \to \dfrac{2x^2-5x+3}{3x^2-7x+4} \geq 0,\ 3x^2-7x+4 \neq 0$

x		1		$\frac{4}{3}$		$\frac{3}{2}$	
$2x^2-5x+3$	$+$	0	$-$		$-$	0	$+$
$3x^2-7x+4$	$+$	0	$-$	0	$+$		$+$
	$+$		$+$	0	$-$	0	$+$

$D_f = (-\infty,+\frac{4}{3}) \cup [\frac{3}{2},+\infty)$