

الف) $f(x) = \sqrt{\frac{x-1}{x}} - \frac{x}{x-1} \geq 0 \rightarrow \frac{(x-1)^2 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{x^2 - 2x + 1 - x^2}{x(x-1)} \geq 0 \rightarrow \frac{-2x+1}{x(x-1)} \geq 0$
 $D_f = (-\infty, 0) \cup [\frac{1}{2}, 1)$

ب) $\frac{\frac{1}{x+1} - \frac{2}{x}}{\frac{2}{x-1} + \frac{1}{x+2}}$
 $x+1 \neq 0 \rightarrow x \neq -1$
 $x \neq 0$
 $x-1 \neq 0 \rightarrow x \neq 1$
 $x+2 \neq 0 \rightarrow x \neq -2$
 $D_f = \mathbb{R} - \{-2, -1, -\frac{1}{2}, 0, 1\}$

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الف) $f(x) = \sqrt{(\frac{1}{x})^2 - 9} (x^2 - x^3) \geq 0$
 $D_f = (-\infty, -2] \cup [\frac{2}{3}, +\infty)$

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ب) $\sqrt{x-1} + \sqrt{y+1} = 3$
 $x \in [1, 10]$
 $y \in [-1, 8]$
 $x-1 \geq 0 \rightarrow x \geq 1$
 $y+1 \geq 0 \rightarrow y \geq -1$
 $y+1 = (3 - \sqrt{x-1})^2 \rightarrow y+1 = (3-x)^2 = 9 \rightarrow y=8$
 $x=10$
 $y \in [-1, 8]$

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$f(x) = \frac{\log_2(x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$
 $D_f = (-\infty, -1) \cup (1, +\infty)$
 $x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) > 0$
 $x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow x > 1, x < -1$
 $\sqrt{x^2 - 1} + 1 \neq 0 \rightarrow \sqrt{x^2 - 1} \neq -1$

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$f(x) = \sqrt{4 + ax - x^2} \geq 0$
 $D_f = [-2, b]$
 $a+b=0 \rightarrow -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$
 $-2+b = -\frac{a}{-1} = a \rightarrow -2 + \frac{3}{2} = a \rightarrow a = -\frac{1}{2}$
 $(-2) \times b = \frac{3}{-1} = -3 \rightarrow -2b = -3 \rightarrow b = \frac{3}{2}$

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$f(x) = \begin{cases} x^2 - 2 & x \geq 1 \\ x^2 + 3 & x < 1 \end{cases}$
 $g(x) = \sqrt{f(x) - x} \geq 0 \rightarrow f(x) - x \geq 0 \rightarrow f(x) \geq x$
 $\textcircled{1} x^2 - 2 \geq x$
 $x^2 - x - 2 \geq 0$
 $x \geq 2$
 $x \geq 1$
 $[2, +\infty)$
 $\textcircled{2} x^2 + 3 \geq x$
 $x^2 - x + 3 \geq 0$
 $x \geq -3$
 $x < 1$
 $\rightarrow [-3, 1)$
 $\textcircled{1} \cup \textcircled{2} = [-3, 1) \cup [2, +\infty) = [-3, +\infty)$
 $D_g = [-3, +\infty)$

$$P(n) = r^n - kn$$

$$P\left(\sin \frac{\pi}{r} + \cos \frac{\pi}{r}\right) =$$

$$n = \frac{\sin \frac{\pi}{r} + \cos \frac{\pi}{r}}{\frac{\sqrt{r}}{r}} \rightarrow x = \frac{\sqrt{r}}{r} + 1$$

$$\begin{aligned} P(n) &= r\left(\frac{\sqrt{r}}{r} + 1\right)^r - k\left(\frac{\sqrt{r}}{r} + 1\right) = \frac{r}{r} + r\sqrt{r} - r\sqrt{r} - k = \frac{-1}{r} \\ \left(\frac{\sqrt{r}}{r}\right)^r + r\left(\frac{\sqrt{r}}{r}\right) + 1 &= \frac{r}{r} + \sqrt{r} + 1 \\ &= \frac{r}{r} + \frac{r}{r} + \sqrt{r} = \frac{r}{r} + \sqrt{r} \end{aligned}$$

$$P(n) = r\left(\frac{\sqrt{r}}{r} + 1\right)^r - k\left(\frac{\sqrt{r}}{r} + 1\right) = \frac{r}{r} + r\sqrt{r} - r\sqrt{r} - k = \frac{-1}{r}$$

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$$P(n) = \frac{n+1}{n-1}$$

$$P(P(a)) = a$$

$$a \in \mathbb{R} - \{1\}$$

$$P(a) = \frac{a+1}{a-1}$$

$$\text{ناتجانسی} \Rightarrow a \neq 1$$

$$P(P(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{\frac{ra}{a-1}}{\frac{r}{a-1}} = a$$

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$$P(n) = n^r - kn$$

$$g(a) = \sqrt{a^r - 1}$$

$$g(n) = \sqrt{n^r - 1}$$

$$P(g(a)) = (\sqrt{a^r - 1})^r - k\sqrt{a^r - 1} = a^r - 1 - k\sqrt{a^r - 1} = -k$$

$$P(g(a)) = -k$$

$$\rightarrow a^r - 1 - k\sqrt{a^r - 1} = -k \rightarrow (a^r + r)^{\frac{r}{2}} = (k\sqrt{a^r - 1})^{\frac{r}{2}}$$

$$a = 9$$

$$\rightarrow (a^r + r)^r = 14(a^r - 1) \rightarrow a^r + 4a^r + 9 = 14a^r - 14$$

$$\rightarrow a^r - 10a^r + 23 = 0 \rightarrow (a^r - 5)^2 = 0 \rightarrow a^r = 5 \rightarrow a = \pm\sqrt{5}$$

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$$P(n+p) = n^r - an + q$$

$$P(k) \times P(-p) = ? \Rightarrow \forall r, k, p = 1 \times 1$$

$$\rightarrow n+p = -r \rightarrow n = -4$$

$$P(-4) = (-4)^r - a(-4) + q = r^4 + 4a + q = 1 \times 1$$

$$\rightarrow n+p = 1 \rightarrow n = 1$$

$$P(1) = 1^r - a(1) + q = 1 - a + q = 1$$

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$$P(t \cos^n + a \cos^n) = \tan^n$$

$$P\left(\frac{r}{r}, \frac{a}{r}\right) = ?$$

$$t \cos^n + a \cos^n = \frac{r}{r} \rightarrow (t \cos^n + a \cos^n - \frac{r}{r} = 0) \rightarrow 1t^r + 1a^r - r = 0$$

$$t a^r = \frac{1}{t^r} - 1 = \frac{1}{(\frac{1}{t})^r} - 1 = \frac{1}{\frac{1}{t^r}} - 1 = t^r - 1 = \frac{r}{r}$$

$$t = \frac{-10 \pm \sqrt{100 + 12t}}{14} = \frac{-10 \pm \sqrt{12t}}{14}$$

$$= \frac{-10 \pm 11}{14} \rightarrow t_1 = \frac{1}{14} = \frac{1}{r} \checkmark$$

$$\rightarrow t_2 = \frac{-10}{14} = -\frac{5}{7} \text{ Use}$$

$$n \cos \rightarrow -10 \text{ Use}$$

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