

۱۹۵ آزمون ۱۳۸۵

۱) $\sqrt{\frac{x-1}{x}} = \frac{x}{x-1} \Rightarrow \frac{x-1}{x} = \frac{x^2}{(x-1)^2} \Rightarrow \frac{-x+1}{x(x-1)} \geq 0 \Rightarrow \frac{-x+1}{x(x-1)} \geq 0$

$\Rightarrow D_f = (-\infty, 0) \cup [1, +\infty)$

۲) $\frac{f(x) - f(x+1)}{(x+1) - x} = \frac{f(x) - f(x+1)}{1} = f(x) - f(x+1)$

$\Rightarrow D_f = \mathbb{R} - \{-\frac{1}{\varepsilon}, -\frac{1}{2}, 0, 1\}$

۱) $\sqrt{\left(\frac{1}{x}\right)^2 - 1} (x^2 - \varepsilon^2) \Rightarrow \frac{-x}{x^2 - 1} \Rightarrow D_f = (-\infty, -1] \cup [\frac{1}{\varepsilon}, +\infty)$

۱.۵

۱.۵ $\sqrt{x+1} + \sqrt{y+1} \geq 2 \Rightarrow y+1 \geq 4 - \sqrt{x+1} \Rightarrow y \geq 3 - \sqrt{x+1}$

$\Rightarrow D_f = x \geq 1$

۳) $\frac{f(x) - f(x+1)}{(x+1) - x} = \frac{f(x) - f(x+1)}{1} = f(x) - f(x+1)$

$\Rightarrow D_f = (-\infty, -1) \cup (1, +\infty)$

$f(x) = \frac{\log(x^2 - x + 1)}{\sqrt{x^2 - 1} + 1} > 0 \Rightarrow (x-1)(x+1) \Rightarrow \frac{-1}{x-1} \Rightarrow D_f = (-\infty, -1) \cup (1, +\infty)$

$\Rightarrow D_f = (-\infty, -1) \cup (1, +\infty)$

$f(x) = \sqrt{x+ax-x^2} \Rightarrow [-r, b]$

$\sqrt{x+(-r)a-\varepsilon} \geq 0 \Rightarrow a \geq \frac{1}{r}$

$\Rightarrow a+b = \frac{3}{r} - \frac{1}{r} = \frac{2}{r}$

$f(x) = \sqrt{x^2 - 2x + 1} \Rightarrow f(x) = |x-1|$

$g(x) = \sqrt{f(x) - x} \Rightarrow f(x) \geq x$

$\Rightarrow [-3, +\infty)$

$f(\sin \frac{\pi}{4} + \cos \frac{\pi}{4}) = ? \Rightarrow \sin \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2} \Rightarrow x = \frac{\sqrt{2}}{2} + 1$

$f(x) = 2x^2 - \varepsilon x \Rightarrow x(2x - \varepsilon) = (\frac{\sqrt{2}}{2} + 1)(\sqrt{2} + \sqrt{2} - \varepsilon) \Rightarrow \frac{3}{2} + \sqrt{2} - \sqrt{2} - \frac{\varepsilon}{2} = \frac{3}{2} - \frac{\varepsilon}{2} = \frac{1}{2}$

$f(f(a)) = a$

$f(x) = \frac{x+1}{x-1} \Rightarrow f(a) = \frac{a+1}{a-1} \Rightarrow f(f(a)) = \frac{\frac{a+1}{a-1} + 1}{\frac{a+1}{a-1} - 1} = \frac{a+1+a-1}{a+1-a+1} = \frac{2a}{2} = a$

$\Rightarrow a = \mathbb{R} \setminus \{1\}$

$f(g(a)) = -\varepsilon \Rightarrow a = ?$

$f(x) = x^2 - \varepsilon x \Rightarrow g(x) = \sqrt{x^2 - 1}$

$x^2 - \varepsilon x = -\varepsilon \Rightarrow x^2 - \varepsilon x + \varepsilon = 0 \Rightarrow (x-\varepsilon)^2 = 0 \Rightarrow x = \varepsilon$

$\Rightarrow g(x) = \sqrt{x^2 - 1} \Rightarrow \sqrt{\varepsilon^2 - 1} = \varepsilon \Rightarrow \varepsilon^2 - 1 = \varepsilon^2 \Rightarrow -1 = 0$

$f(x+y) = x^2 - \omega x + y$

$f(-y) = 6 \Rightarrow (-y)^2 - \omega(-y) + y = 6 \Rightarrow y^2 + \omega y + y = 6 \Rightarrow y^2 + (\omega+1)y - 6 = 0$

$\sqrt{y^2 + (\omega+1)y - 6} = -y \Rightarrow y^2 + (\omega+1)y - 6 = y^2 \Rightarrow (\omega+1)y - 6 = 0 \Rightarrow y = \frac{6}{\omega+1}$

$f(1) = 2 \Rightarrow 1^2 - \omega(1) + 1 = 2 \Rightarrow 2 - \omega = 2 \Rightarrow \omega = 0$

$2 \times \sqrt{2} = f(-y) \times f(1) = 1 \times 2 = 2$

$f(\cos x + \omega \cos x) = \tan^2$

$f(x) = ?$

$\Rightarrow \varepsilon \cos^2 x + \omega \cos x = 3, 0$

$\varepsilon \cos^2 x + \omega \cos x - 3 = 0$

$\Rightarrow \cos^2 x + \frac{\omega}{\varepsilon} \cos x - \frac{3}{\varepsilon} = 0$

$(\cos x + \frac{\omega}{2\varepsilon})(\cos x - \frac{\omega}{2\varepsilon}) = 0$

$\Rightarrow \cos x = \frac{\omega}{2\varepsilon} \Rightarrow \frac{1}{\varepsilon} = \cos x \Rightarrow \frac{1}{\varepsilon} = \cos \frac{\pi}{2} = 0$

$\Rightarrow \tan^2(\frac{\pi}{2}) = (\sqrt{3})^2 = 3$

ب- ۲) $\sqrt{x-1} + \sqrt{y+1} = 3$, $\sqrt{y+1} = 3 - \sqrt{x-1} \rightarrow \sqrt[3]{x-1} \geq 0 \rightarrow \sqrt{x-1} \leq 3 \rightarrow x \leq 10$ $\mathbb{I}$

$\downarrow$
 $x-1 \geq 0 \rightarrow x \geq 1$ $\mathbb{I}$

$\mathbb{I}, \mathbb{II} \rightarrow 1 \leq x \leq 10 \rightarrow D_f = [1, 10]$