

$f(x, t) = \begin{cases} x+1 & x > 1 \\ x-1 & x < 1 \end{cases} \xrightarrow{x=t, t \rightarrow x=t-1} \begin{cases} t-2 > 1 \rightarrow t > 3 \rightarrow f(t-2) = t-1 \\ t-2 < 1 \rightarrow t < 3 \rightarrow f(t-2) = t-3 \end{cases}$
 $\Rightarrow f(t) = \begin{cases} t+1 & t > 3 \\ t-3 & t < 3 \end{cases} \rightarrow f(x) = \begin{cases} x+1 & x > 3 \\ x-3 & x < 3 \end{cases}$
 $f(3-x) \rightarrow \begin{cases} 3-x < 3 \rightarrow x > 0 \rightarrow f(3-x) = 3-x \\ 3-x > 3 \rightarrow x < 0 \rightarrow f(3-x) = 1-3x \end{cases}$
 $f(x+1) \rightarrow \begin{cases} x+1 < 3 \rightarrow x < 2 \rightarrow f(x+1) = x-1 \\ x+1 > 3 \rightarrow x > 2 \rightarrow f(x+1) = 1-3(x+1) \end{cases}$

$\sqrt{f(3-x)} - f(x+1) \rightarrow f(3-x) - f(x+1) > 0$
 $x > 2 \rightarrow 3-x - (x-1) = 1-2x > 0 \rightarrow x < \frac{1}{2}$
 $0 < x < \frac{1}{2} \rightarrow 3-x - (x+1) = 2-2x > 0 \rightarrow x < 1$
 $\rightarrow I \cap II \rightarrow (0, \frac{1}{2}] \rightarrow I$
 $x < 0 \rightarrow 1-3x - (x+1) = -4x > 0 \rightarrow x < \frac{1}{4}$
 $\rightarrow (-\infty, 0] \cap I \rightarrow (-\infty, \frac{1}{4}]$

$f(x) = (x^r + a)^r$
 $f'(x) = r(x^r + a)^{r-1} \cdot (rx^{r-1})$
 $f''(x) = r(r-1)(x^r + a)^{r-2} \cdot (rx^{r-1})^2 + r(x^r + a)^{r-1} \cdot (r(r-1)x^{r-2})$
 $f''(x) < f''(x') \rightarrow (r(r-1)(x^r + a)^{r-2} \cdot (rx^{r-1})^2 + r(r-1)x^{r-2}(x^r + a)^{r-1}) < (r(r-1)(x'^r + a)^{r-2} \cdot (rx'^{r-1})^2 + r(r-1)x'^{r-2}(x'^r + a)^{r-1})$
 $\rightarrow x < 0$

$y = |x| \left(x^{\frac{1}{2}} + \frac{1}{x} \right)$
 $\rightarrow x > 0 \rightarrow y = x \left(x^{\frac{1}{2}} + \frac{1}{x} \right) \rightarrow y = x^{\frac{3}{2}} + 1$
 $\rightarrow x < 0 \rightarrow y = -x \left(x^{\frac{1}{2}} + \frac{1}{x} \right) \rightarrow y = -x^{\frac{3}{2}} - 1$

غیر یکپارچه

$f(x) = \frac{x+1}{|x|-|x+1|}$
 $x > 0 \rightarrow \frac{x+1}{x-1-x-1} = \frac{x+1}{-2x-2} = -\frac{x+1}{2(x+1)} = -\frac{1}{2}$
 $-1 < x < 0 \rightarrow \frac{x+1}{-x-x-1} = \frac{x+1}{-2x-1} = \frac{x+1}{-1(x+1)} = -1$
 $x < -1 \rightarrow \frac{x+1}{-x+x+1} = \frac{x+1}{1} = x+1$

$\boxed{0 = 0}$

 تابع از $(-\infty, 0]$ صعودی است

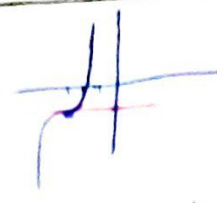
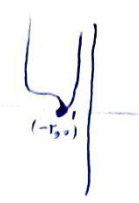
$y = \log_{\frac{1}{2}}(x^2 - 2x - 1) \rightarrow x^2 - 2x - 1 > 0 \rightarrow (x-4)(x+2) > 0 \rightarrow x < -2 \text{ or } x > 4$

$y = \log_{\frac{1}{2}} x \rightarrow 0 < x < 1 \rightarrow$

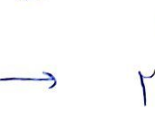
$y = \log_{\frac{1}{2}}(x^2 - 2x - 1) \rightarrow x > 4 \rightarrow$
 $x < -2 \rightarrow$

تابع $(-2, 4)$ آلیه صعودی است

$f(x) = (x^2 - 2)^n$
 $\rightarrow t^n \rightarrow t > 1 \rightarrow$  $\rightarrow$ $x^2 - 2 > 1 \rightarrow x^2 > 3 \rightarrow x > \sqrt{3}$ 
 $\rightarrow t^n \rightarrow t < 1 \rightarrow$  $\rightarrow$ $x^2 - 2 < 1 \rightarrow x^2 < 3 \rightarrow x < \sqrt{3}$ 
 $C_D \rightarrow (-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$
 $f(x) = (x^2 - 2)^{n/2} \rightarrow x^2 - 2 > 1 \rightarrow x^2 > 3 \rightarrow x^2 - 2 < 0 \rightarrow$ 
 $C_D \rightarrow (-\infty, -\sqrt{2}) \cup (\sqrt{2}, \infty)$

$f(x) = (x^2 - 2)^3 - 1$  $\rightarrow$ $(x^2 - 2)^3 - 1 = x^6 + 27x^4 + 9x^2 + 27x - 1 = x^6 + 9x^4 + 27x^2 + 27x - 1$
 $|x^2 - 2|$  $\rightarrow$ $(-2, 2) \rightarrow$ $[2, -2]$
 $\rightarrow$ $(-2, 2)$ $\rightarrow$ $[2, -2]$

$f(x) = x \cdot \sqrt{x} - 2 \rightarrow f(x) \leq f(\sqrt{x}) \rightarrow f(x) \leq \sqrt{x}$
 $x \cdot \sqrt{x} - 2 \leq \sqrt{x} \rightarrow x - 2 \leq \frac{1}{\sqrt{x}} \rightarrow x \leq 2$
 $\sqrt{x} \rightarrow x \geq 0$ II
 $I \cup II \rightarrow 0 \leq x \leq 2 \rightarrow \{0, 1, 2\}$

$f(x) = \sqrt[3]{9x^2 - 1}$ 
 $k \in \mathbb{Z}$
 $x = 1 < \pi \rightarrow$ 
 $x = k\pi + \frac{\pi}{2} \rightarrow$ 
 $R_f = [-\frac{\pi}{2}, \frac{\pi}{2}] \rightarrow \frac{18}{\pi} - (-\frac{\pi}{2}) \rightarrow \frac{18}{\pi} + \frac{\pi}{2} \rightarrow \frac{18}{\pi} + \frac{\pi}{2} = \frac{37}{2}$

$g(x) = \frac{x - [x, 2]}{2} = \frac{[x, 2] - x}{2}$