پویا نظلیان — به نام خدا — ۱۹ آذر — ۱۱

DATE / / SUBJECT:

① (۲)

$D_{gof(x)} = \{x \mid x\in D_f \wedge f(x)\in D_g\}$ $\quad D_{f(x)}: 0 \le \log_2^{x-1} \to 2 \le x \cap 1<x$ ، $2\le x$ (تعریف $\log_2$)

$D_{g(x)}: 0 \le 1-(x-2)^2 \to x=2 \to \log_2^{x-1}=4 \to x=17 \to [2,+\infty)\cap\{17\}=\{17\}$

$\to D_{gof(x)}=\{17\}$ ✓

② (۲)

$fog(x)=2x^2-6x+11 \;,\; gof(x)=4x^2-20x+25-6x+15+8=4x^2-26x+48$

$\to 2x^2-6x+11=4x^2-26x+48 \to 2x^2-20x+37=0 \to$ ریشه‌ها: $\alpha,\beta \to \alpha^2+\beta^2=S^2-2P \to$

$S=10 \;,\; P=\frac{37}{2} \to S^2-2P=100-37=63$ ✓

③ (۲)

$f(x)=2x=k \to x=\frac{k}{2} \to g(k)=\frac{5k^2}{4}+11 \to g(x-7)=\frac{5(x-7)^2}{4}+11 \to$

$\text{Min}\; g(x-7)=11$ ✓

④ (۲)

$3g(x)-4=3x^2-6x-1 \to g(x)=x^2-2x+1=(x-1)^2 \to gof(x)=(2x-5)^2$ ✓

⑤ (۲)

$(f-g)(x)=-x+5 \to$ $f(x)-g(x)=-x+5 \to 2f(x)-2g(x)=-2x+10$

$\to 2f(x)-(3f(x)-14)=-f(x)+14=-2x+10 \to$

$fog(x)=-\frac{2}{a}x+2$ $\quad f(x)=2x+4 \to 2g(x)+4=-\frac{2}{a}x+2 \to$

$g(x)=\left(-\frac{2}{a}x-2\right)\times\frac{1}{2}$

$g(x)=3x-1$

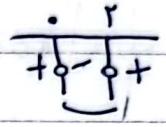
$\to -\frac{1}{a}x-1=3x-1 \to -\frac{1}{a}=3 \to a=-\frac{1}{3}$ ✓

Sahand

پویانمایان

DATE / / SUBJECT:

$f \circ g(x) = 9^{\log_3 x^2} = x^2 ; 0 < x$, $g \circ f(x) = 2x \rightarrow x^2 - 2x \leq 0 \rightarrow$ (۹)



(۲)

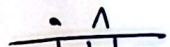
$\rightarrow [0,2] \cap (0,+\infty) = [0,2] \rightarrow x \in \mathbb{Z} \rightarrow x = 1, 2$ ارفاقی $\rightarrow$ ۳ عدد صحیح

$f(x - \frac{2}{x}) = x^2 - 4 + \frac{4}{x^2} + x - \frac{2}{x} \rightarrow f(x - \frac{2}{x}) = (x - \frac{2}{x})^2 + x - \frac{2}{x} \rightarrow x - \frac{2}{x} = A$ (۷)

(۲)

$f(A) = A^2 + A \rightarrow f(x) = x^2 + x$ ✓

$D_{f \circ g(x)} = \{ x \mid x \in D_g \wedge g(x) \in D_f \} \Rightarrow D_f = (0, +\infty) , D_g = \mathbb{R} \rightarrow$ (۸)

$0 < 8x - x^2 \rightarrow$  $\rightarrow (0,8) \cap \mathbb{R} = (0,8) = D_{f \circ g(x)} = A$ (۲)

$f \circ g(x) = \log_2^{8x - x^2} \rightarrow \max_{\text{ماکزیمم}} 8x - x^2 \xrightarrow{x=4}$ (۱۶) $\rightarrow \log_2^{16} = 4 \rightarrow R_{f \circ g} = (-\infty, 4] = B$ ✓

$\rightarrow A \cap B = (-\infty, 4] \cap (0,8) = (0,4] \rightarrow x \in \mathbb{Z} \rightarrow x = \{1, 2, 3, 4\}$

$f(x) = k \rightarrow f(k) = 1 - 2k^2 \rightarrow f(x) = 1 - 2x^2$ (۹)

$h(g(x)) = (x-1)^2 + 2x - 1 = 1 - 2x^2 \rightarrow -2x^2 - 2x + 2 = x^2 - 2x^2 + 2x - 1 \rightarrow$ (۲)

$x^2 + 4x - 3 = 0 \rightarrow$ یک ریشه ✓

$g(2) = 8 \rightarrow f(x) = 2 \rightarrow x = -1 \rightarrow -a - 2 = 8 \rightarrow a = -10$ (۱۰)

(۱)

$x - \frac{4}{x} = 3 \quad x^2 - 3x - 4 = 0$

$x < 2 \rightarrow 4 + 2a + 8 = 8 \rightarrow a = 0$

$x < -1$ ✓

Sahand