نوید امینی - تکلیف ۸

(۱)

$D_{g\circ f} = \{D_f \mid f(x) \in D_g\}$

$D_f \to \sqrt{\log_2^{(x-1)}} \geq 0 \to x>1$

$\log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2$

$\Big\} \cap \to x \geq 2$

$g(x) = \sqrt{-(x-2)^2} \to D_g = \{2\}$ $\sqrt{\log_2^{x-1}} = 2 \to \log_2^{x-1} = 4 \to x-1=16 \to x=17 \to D_{g\circ f} = \{17\}$

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(۲)

$f\circ g = 2(x^2-3x+8)-5 = 2x^2-6x+16-5 = 2x^2-6x+11$

$g\circ f = (2x-5)^2-3(2x-5)+8 = 4x^2-20x+25-6x+15+8 = 4x^2-26x+48$

$f\circ g = g\circ f \to 2x^2-6x+11 = 4x^2-26x+48 \to 2x^2-20x+37=0$ $\alpha^2+\beta^2 = S^2-2P$

$(\frac{20}{2})^2 - \cancel{2}(\frac{37}{\cancel{2}}) = 100-37 = 63$

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(۳)

$g\circ f(x) = g(2x) = 5x^2+11$
$2x=t \to x=\frac{t}{2}$
$\Big\} \to g(t) = 5(\frac{t}{2})^2+11 = \frac{5t^2}{4}+11 \to g(x) = \frac{5x^2}{4}+11$

$g(x-7) = \frac{5(x-7)^2}{4}+11 = \frac{5}{4}\underbrace{(x^2-14x+49)}_{min=0}+11 \to \min g(x-7) = 11$

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(۴)

$f(g(x)) = 3g-4 = 3x^2-6x-1 \to \frac{3x^2-6x+3}{3} = g(x) = x^2-2x+1 = (x-1)^2$

$g\circ f(x) = (3x-4-1)^2 = (3x-5)^2 = 9x^2-30x+25$

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(۵)

$f-g = -x+5$

$f\circ g = Ax+2$

$g(x) = a'x+b'$ $(a-a')x+b-b' = -x+5 \to a-a' = -1$

$f(x) = ax+b$ $b-b' = 5$

$2a'x+2b' = 3ax+3b-14 \to \begin{cases} 3a = 2a' \\ a-a' = -1 \end{cases}$ $3(a'-1) = 2a' \to a' = 3$
$a = 2$

$2b' = 3b-14$

$\frac{2b-3b+14}{2} = 5 \to -b = 10-14 \to b = 4,\ b' = -1$ $a=-3$

$f(x) = 2x+4$
$g(x) = 3x-1$
$\to f\circ g(x) = 2(3x-1)+4$

$6x-2+4 = 6x+2 = 0 \to x = -$

(6)

$fog = 9^{\log_3^x} = x^2 \ , \ x>0$

$gof = \log_3^{9^x} = 2x \quad x \in \mathbb{R}$

$x^2 \leq 2x \to x^2 - 2x \leq 0$

$x(x-2) \leq 0 \to$ $\begin{array}{c|c|c|c} & 0 & & 2 \\ + & & - & & + \end{array}$ $x \in [0,2]$

$\xrightarrow{\cap} x \in (0,2]$ $\quad x_{\text{طبیعی}} = \{1,2\}$

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(7)

$f(x-\frac{2}{x}) = x^2 + \frac{4}{x^2} - 4 + x - \frac{2}{x}$

$(x-\frac{2}{x})^2 = x^2 + \frac{4}{x^2} - 4$

$f(x-\frac{2}{x}) = (x-\frac{2}{x})^2 + (x-\frac{2}{x}) \Rightarrow f(t) = t^2 + t$

$\boxed{f(x) = x^2 + x}$

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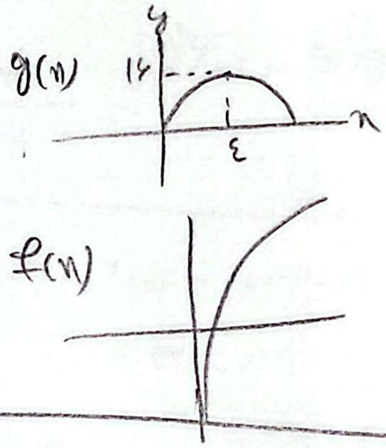
(8)

$fog = \log_2^{8x - x^2}$

$D_{fog} \to x(8-x) > 0$ $\quad \begin{array}{c|c|c|c} & 0 & & 8 \\ - & & + & & - \end{array}$ $x \in (0,8)$

$R_{fog} = (-\infty, \log_2^{16}] = (-\infty, 4]$



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(9)

$fof(x) = 1 - 3f^2(x) \xrightarrow{x = f^{-1}} fofof^{-1} = 1 - 3(f(f^{-1}) \times f(f^{-1})) \to f(x) = 1 - 3x^2$

$hog(x) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x \cancel{-1} + 2x - 2 \cancel{+1} = x^3 - 3x^2 + 5x - 2$

$hog = f \to 1 - \cancel{3x^2} = x^3 \cancel{- 3x^2} + 5x - 2 \to x^3 + 5x - 3 = 0$

یکبار محور را قطع می‌کند ← اکیداً صعودی ← $y' = 3x^2 + 5 > 0$ ← (صعودی

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(10)

$g(x - \frac{4}{x}) = ax^3 + 2x \to$

$x - \frac{4}{x} = 3 \to x^2 - 3x - 4 = 0 \quad (x-4)(x+1) = 0 \quad x = 4, -1$

$x = 4 \to g(3) = 8 = 16a + 8 \to a = 0$

$x = -1 \to g(3) = 8 \quad -a - 2 = 8$

$a = -10$

نوروزی