نام و نام خانوادگی .... امیرحمید ابوطالبی .... **پاسخنامه تشریحی تکلیف شماره .... کلاس ....**

$g\circ f(x) = \sqrt{-(\sqrt{\log_2(x-1)})^2 + 4\sqrt{\log_2(x-1)} - 4} = \sqrt{-(\sqrt{\log_2(x-1)} - 2)^2}$ $\therefore x-1>0 \to x>1$ $\log_2^{x-1} \geq 0 \to x-1 \geq 1 \to x \geq 2$ $\sqrt{\log_2^{x-1}} = 2 \to \therefore \log_2^{x-1} = 4 \to \boxed{x-1 = 16 \to x = 17}$	۱
$f\circ g(x) = 2x^2 - 6x + 16 - 5 = 2x^2 - 6x + 11$ $g\circ f(x) = (2x-5)^2 - 3(2x-5) + 8 = 4x^2 + 25 - 20x - 6x + 15 + 8 = 4x^2 - 26x + 48$ $f\circ g(x) = g\circ f(x) \longrightarrow 2x^2 - 6x + 11 = 4x^2 - 26x + 48 \Rightarrow 2x^2 - 20x + 37 = 0 \to \boxed{\alpha, \beta}$ ریشه ها $\alpha^2 + \beta^2 = S^2 - 2P \to 100 - 37 = \boxed{63}$	۲
$g\circ f(x) = 5x^2 + 11 \longrightarrow g(2x) = 5x^2 + 11$ $f(x) = 2x$ $g(2x) = 5x^2 + 11 \xrightarrow{x = \frac{t}{2} \to t = 2x} g(t) = 5\left(\frac{t}{2}\right)^2 + 11 \longrightarrow g(t) = \frac{5}{4}t^2 + 11$ $t = x-7 \longrightarrow g(x-7) = \frac{5}{4}(x-7)^2 + 11 \longrightarrow x = 7 \longrightarrow \boxed{g(x-7) = 11}$ کمترین مقدار تابع	۳
$f(x) = 3x - 4$ $f\circ g(x) = 3x^2 - 6x - 1$ $g(x) = x^2 - 2x + 1 = (x-1)^2$ $g\circ f(x) = (3x-4-1)^2 = (3x-5)^2 = \boxed{9x^2 + 25 - 30x}$	۴
$2g(x) = 3f(x) - 14$ $g(x) = \frac{3}{2}f(x) - 7$ $f(x) - g(x) = -x + 5 \longrightarrow f(x) - \frac{3}{2}f(x) + \cancel{7}^{2} = -x + \cancel{5} \to -\frac{1}{2}f(x) = -x - 2$ $\boxed{f(x) = 2x + 4}$ $g(x) = 3x - 1$ $f\circ g(x) = 2(3x-1) + 4 = 6x + 2 \longrightarrow f\circ g(a) = 0 \to 0 = 6a + 2$ $\boxed{a = -\frac{1}{3}}$	۵

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$f(x)=9^x$

$g(x)=\log_3 x \longrightarrow x>0 \to I$

$fog(x)=9^{(\log_3 x)}=x^2$

$gof(x)=\log_3 9^x=2x$

$fog(x)\leqslant gof(x) \longrightarrow x^2\leqslant 2x \longrightarrow x^2-2x\leqslant 0 \to \;\; + \;\; 0 \;\; - \;\; 2 \;\; +$

$\boxed{0\leqslant x\leqslant 2}$

$\to 0,1,2 \longrightarrow \therefore \boxed{1,2}$ ۲ عدد

می تواند باشد طبق شرط ①

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$f\left(\frac{x^2-2}{x}\right)=x^2+x-4-\frac{2}{x}+\frac{4}{x^2} \longrightarrow f\left(\frac{x^2-2}{x}\right)=x^2+\frac{4}{x^2}-4+x-\frac{2}{x}$

$f\left(x-\frac{2}{x}\right)=x^2+\frac{4}{x^2}-4+x-\frac{2}{x} \longrightarrow f\left(x-\frac{2}{x}\right)=\left(x-\frac{2}{x}\right)^2+x-\frac{2}{x} \xrightarrow{(x-\frac{2}{x})=t} f(t)=t^2+t$

$\boxed{f(x)=x^2+x}$

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$f(x)=\log_2 x$

$g(x)=8x-x^2$

$fog(x)=\log_2^{8x-x^2} \longrightarrow 8x-x^2>0 \to \;\; - \;\; 0 \;\; + \;\; 8 \;\; - \;\; \to 0<x<8$

$\boxed{D_{fog}=(0,8)\to A}$

$R_{fog} \to 8x-x^2 \to -\frac{b}{2a}=\boxed{4} \to \max(8x-x^2)=16$

$\min(8x-x^2)=0 \to 0<x<8$

$\to 0<()\leqslant 16$

$\log_2^{(8x-x^2)} \longrightarrow \boxed{R_{fog}=(-\infty,4]\to B}$

$A\cap B=(0,4]$

۱، ۲، ۳، ۴ ← ۴ عدد

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$fof(x)=1-3f^2(x) \longrightarrow f(f(x))=1-3f^2(x) \xrightarrow[f(x)=y]{} f(y)=1-3y^2 \longrightarrow \boxed{f(x)=1-3x^2}$

$g(x)=x-1$

$h(x)=x^3+2x+1$

$h(g(x))=(x-1)^3+2(x-1)+1=x^3-\cancel{1}-3x^2+3x+2x-2+\cancel{1}=x^3-3x^2+5x-2$

$h(g(x))=f(x) \longrightarrow x^3-3x^2+5x-2=1-3x^2 \longrightarrow \cancel{x^3-3x^2+5x-3=0} \quad x^3+5x-3=0$

۱ ریشه دارد

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$f(x)=x-\frac{4}{x}$

$gof(x)=ax^3+2x$

$g\left(x-\frac{4}{x}\right)=ax^3+2x$

$x-\frac{4}{x}=3 \to x^2-4=3x \to x^2-3x-4=0 \to (x-4)(x+1)=0 \to x=4,\ -1$

$\xrightarrow{x=4} g(3)=64a+8=8 \longrightarrow \boxed{a=0} \qquad x=-1 \longrightarrow g(3)=-a-2=8 \to \boxed{a=-10}$