نام و نام خانوادگی ...آرین غضنفری سیادکوهی... پاسخنامه تشریحی تکلیف شماره ...۸... کلاس ...۱۱۲م B...

۱

$D_{gof} = \{x \in D_f , f(x) \in D_g\} \Rightarrow f(x) = \sqrt{\log_2^{(x-1)}}$ $\nearrow x-1>0 \Rightarrow x>1$ $\searrow \log_2^{x-1} \geq 0 \Rightarrow x \geq 2$ $\cap \Rightarrow D_f = [2,\infty)$

$g(x) = \sqrt{-x^2+4x-4} \Rightarrow g(x) = \sqrt{-(x-2)^2} \Rightarrow D_g = \{2\} \Rightarrow \sqrt{\log_2^{(x-1)}} = 2 \Rightarrow \log_2^{x-1} = 4 \Rightarrow x-1 = 16$

$x = 17$ $D_{gof} = \{17\}$

۲

$f(x) = 2x-5 , g(x) = x^2-4x+8 , fog(x) = f(x^2-4x+8) = 2(x^2-4x+8)-5 = 2x^2-6x+11$

$g(f(x)) = g(2x-5) = (2x-5)^2 - 4(2x-5) + 8 = 4x^2-26x+48$

$gof(x) = fog(x) \Rightarrow 4x^2-26x+48 = 2x^2-6x+11 \Rightarrow 2x^2-20x+37 =$

$\alpha^2+\beta^2 = S^2-2P = 10^2 - 2 \times \frac{37}{2} = 100-37 = 63$

$S = 10 , P = \frac{37}{2}$

۳

$gof(x) = g(2x) = 5x^2+11 , 2x = t \Rightarrow x = \frac{t}{2} \Rightarrow 5x^2+11 = \frac{5t^2}{4} + 11 = g(t)$

$g(x) = \frac{5x^2}{4} + 11$

$g(x-7) = \frac{5(x-7)^2}{4} = \frac{5x^2-70x+245}{4} + 11 = \frac{5x^2-70x+289}{4}$

راس: $x_{ext} = \frac{-b}{2a} = \frac{70}{10} = 7$

$5 \times 7^2 - 70 \times 7 + 289 = 44$ $\frac{44}{4} = 11$

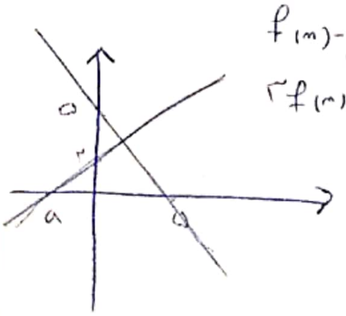
۴

$f(x) = 3x-4 , f(g(x)) = 3x^2-6x-1 \Rightarrow 3y-4 = 3x^2-6x-1 \Rightarrow 3y = 3x^2-6x+3$

$f(g) = 3g-4$ $y = x^2-2x+1$

$g(f(x)) = (3x-4)^2 - 2(3x-4) + 1 = 9x^2-30x+25$

۵



$f(x) - g(x) = -x-5 \xrightarrow{\times 2} 2f(x) - 2g(x) = -2x+10$

$3f(x) - 2g(x) = 12$ $\quad - \quad 3f(x) - 2g(x) = 12$

$-f(x) = -2x-4 \Rightarrow f(x) = 2x+4$

$f(g(x)) = 2(2x-1)+4 = 4x+2$ $g(x) = 2x-1$

$-\frac{2}{a} = -6 \Rightarrow a = -\frac{1}{3}$

| | |
|---|---|
| ۶ | $f(x)=9^x \,,\, g(x)=\log_3^x \,,\, fog(x)=9^{\log_3^x}=x^{\log_3^9}=x^2$ ← $x>0$ ۰ در دامنه تابع نیست<br>$gof(x)=\log_3^{9^x}=2x \quad gof(x)\geq fog \quad 2x\geq x^2 \to 2x-x^2\geq 0$<br>$\begin{array}{ccc} & 0 & & 2 & \\ - & \vert & + & \vert & - \end{array}$<br>$(0,2]$<br>۱ و ۲ گزاره صحیح |
| ۷ | $f\Big(\underbrace{\frac{x^2-2}{x}}_{t}\Big)=x^2+x-4-\frac{2}{x}+\frac{4}{x^2}=\left(x-\frac{2}{x}\right)^2+x-\frac{2}{x}\to f(t)=t^2+t$<br>$x-\frac{2}{x}=t$<br>$f(x)=x^2+x$ |
| ۸ | $f(x)=\log_2^x \,,\, g(x)=8x-x^2 \,,\, fog(x)=f(g(x))=f(8x-x^2)=\log_2^{8x-x^2}$<br>$8x-x^2>0 \longrightarrow x(8-x)>0 \to \begin{array}{ccc} & 0 & & 8 & \\ - & \vert & + & \vert & - \end{array} \to D_{fog}=(0,8)$ ۱<br>رأس سهمی $\to \frac{-b}{2a}=\frac{-8}{-2}=4 \to 8\times 4-4^2=16 \to \log_2^{16}=4 \to R_{fog}=(-\infty,4)$ ۳<br>$۱\cap۳=(0,4] \to$ ۱ ۲ ۳ ۴ گزاره صحیح |
| ۹ | $h(x)=x^3+2x+1 \,,\, g(x)=x-1 \,,\, f(f(x))=1-3f^2(x)\to f(x)=1-3x^2$<br>$h(g(x))=h(x-1)=(x-1)^3+2(x-1)+1=x^3-3x^2+5x-2$<br>$x^3-3x^2+5x-2=1-3x^2 \to x^3+5x-3=0 \longrightarrow$ معادله ۱ ریشه دارد، |
| ۱۰ | $f(x)=x-\frac{4}{x} \,,\, g(f(x))=ax^3+2x \longrightarrow g\Big(x-\frac{4}{x}\Big)=ax^3+2x$<br>$x-\frac{4}{x}=3 \to x^2-3x-4=0 \to x=-1 \,,\, x=4$<br>if $x=-1 \to g(3)=-a-2=8 \to a=-10$<br>if $x=4 \to g(3)=64a+8=8 \to a=0$ |