① $g(x)=\sqrt{-x^2+4x-4}$, $f(x)=\sqrt{\log_2(x-1)}$, $D_{gof(x)}$

x^2-4x+4

$-(x-2)^2 \to$ فقط ۲ قرار می‌گیرد

۱۸

$\sqrt{\log_2(x-1)}=2 \to \log_2^{x-1}=4$ ✓ ②

$\to x-1=16$

$x=17$

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② $f(x)=2x-5$ $g(x)=x^2-3x+8$ $fog(x)=gof(x)$ ②

$fog=2(x^2-3x+8)-5=2x^2-6x+16-5=2x^2-6x+11$

$gof=(2x-5)^2-3(2x-5)+8=4x^2-26x+48$

$2x^2-6x+11=4x^2-26x+48 \to 2x^2-20x+37=0$

$\alpha^2+\beta^2=S^2-2P=100-37=$ ~~53~~ ۶۳

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③ $gof(x)=5x^2+11$ $f(x)=2x$ $\min g(x-7)$

$g(2x)=5x^2+11 \to$ $t=\frac{x}{2} \to gof=5(\frac{t}{2})^2+11 \to \frac{5}{4}x^2+11$ ②

$-\frac{b}{2a}=\frac{-0}{\frac{5}{2}}=0 \to g(0)=11=\min$ ✓ ⑪

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④ $f(x)=3x-4$ $fog(x)=3x^2-6x-1$ $gof(x)=?$ ②

$3g-4=3x^2-6x-1 \to g=\frac{3x^2-6x+3}{3}=g(x)=x^2-2x+1=(x-1)^2$

$gof(x)=(3x-4-1)^2=(3x-5)^2=9x^2-30x+25$ ✓ $9x^2-30x+25$

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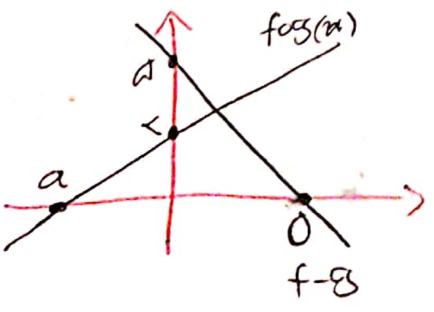
⑤ fog $f-g$ $2g(x)=3f(x)-14$



درست $\to$ $f(x)=2x+4$

$g(x)=3x-1$

$fog(x)=6x+2$

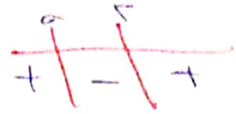
$-\frac{2}{a}x+2=6x+2 \to a=-\frac{1}{3}$

$2g(x)-3f(x)=-14$

$f(x)-g(x)=-x+5$

۶) $f(x)=9^x \quad g(x)=\log_3^x \quad fog(x)\leq gof(x)$ تعداد اعداد صحیح جواب

$fog(x)=9^{\log_3^x}=x^{\log_3^9}=x^2$ ($x>0$) $\quad gof(x)=\log_3^{9^x}=2x \quad x\in\mathbb{R}$ — ۲

$x^2\leq 2x \to x^2-2x\leq 0 \to x(x-2)\leq 0 \to$  $x\in[0,2]$

$(0,2] \to 1, 2 \qquad \{1,2\}$ ✓

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۷) $f\left(\frac{x^2-2}{x}\right)=x^2+x-4-\frac{2}{x}+\frac{4}{x^2} \qquad f(x)=?$ — ۲

$f\left(x-\frac{2}{x}\right)=x^2+\frac{4}{x^2}-4+x-\frac{2}{x}$ $\quad f\left(x-\frac{2}{x}\right)=\left(x-\frac{2}{x}\right)^2+\left(x-\frac{2}{x}\right)\to f(x)=x^2+x$

$\left(x-\frac{2}{x}\right)^2=x^2+\frac{4}{x^2}-4$ $\quad$ ✓ $f(x)=x^2+x$

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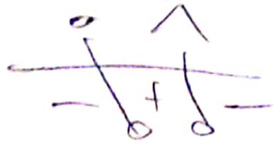
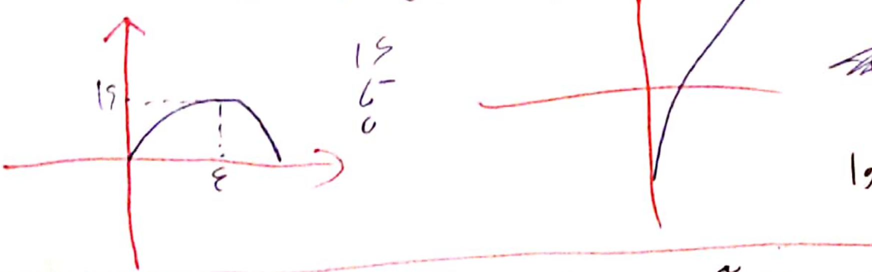
۸) $f(x)=\log_2^x \quad g(x)=8x-x^2 \quad D_{fog}, R_{fog} \quad A\cap B=?$ عضو صحیح

$x(8-x)$ $\quad D_{fog}\to A,\ R_{fog}\to B$



$D_{fog}=(0,8)=A$



$(-\infty,4]=B$ — ۲

$1,2,3,4$ ✓ ۴ عضو

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۹) $fof(x)=1-3f^2(x) \quad g(x)=x-1 \quad h(x)=x^3+2x+1$

$h(g(x))=f(x)$ تعداد ریشه

$(x-1)^3+2(x-1)+1 \quad = \quad f(x)=1-3x^2$

$x^3-3x^2+3x-1+2x-2+1=x^3-3x^2+5x-2$ — ~~۲~~

$x^3-3x^2+5x-2=1-3x^2\to x^3+5x-3=0$ ✓ ۱

$x^3+5x-3=1-3x^2\to x^3+3x^2+5x-4=(x+1)^3+(2x-5)\to$ ابر قطع می کند

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۱۰) $a=? \quad g(3)=8 \quad gof(x)=ax^3+2x \quad f(x)=x-\frac{4}{x}$ — ۲

$f(x)=x-\frac{4}{x}=3\to x^2-4=3x\to x^2-3x-4\to(x-4)(x+1)\Rightarrow x=4,\ -1$

$x=4 \to 64a+8=8 \to a=0$

$x=-1 \to -a-2=8 \to a=-10$

✓ $\{0,10\}$