به نام خدا

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دوازدهم تجربی B

تکلیف ۸

۱- $D_{g\circ f} = \{x \mid x \in D_f \,,\, f(x) \in D_g\}$

$f(x) = \sqrt{\log_2^{x-1}} \Rightarrow \begin{cases} x-1>0 \Rightarrow x>1 & I \\ \log_2^{x-1} \geq 0 \Rightarrow x-1 \geq 1 \Rightarrow x \geq 2 & II \end{cases} \xrightarrow{I \cap II} x \geq 2$

$g(x) = \sqrt{-x^2+4x-4} = \sqrt{-(x-2)^2} \Rightarrow x=2 \Rightarrow \sqrt{\log_2^{x-1}} = 2$

$\Rightarrow \log_2^{x-1} = 4 \Rightarrow x-1 = 16 \Rightarrow \boxed{x=17}$

$D_{g\circ f} = \{17\}$

۲- $f\circ g(x) = 3g-5$, $g\circ f(x) = f^2-2f+8$ $\xrightarrow{f\circ g(x) = g\circ f(x)} 3g-5 = f^2-2f+8$

$\Rightarrow 3x^2-6x+11 = 4x^2+25-20x-6x+15+8 = 4x^2-26x+48$

$\Rightarrow 2x^2-20x+37=0 \Rightarrow \alpha^2+\beta^2 = S^2-2P = 100-37 = 63$

$S = \frac{-b}{a} = \frac{20}{2} = 10$

$P = \frac{c}{a} = \frac{37}{2}$

۳- $g\circ f(x) = g(f(x)) = g(\underbrace{2x}_{t}) = 5x^2+11$

$g(t) = \frac{5}{4}t^2+11 \Rightarrow g(x) = \frac{5}{4}x^2+11$

$\Rightarrow g(x-7) = \frac{5}{4}(x-7)^2+11 \xrightarrow{x=7} g_{min} = 11$

۴- $f\circ g(x) = f(g(x)) = 3g-4 = 3x^2-6x-1 \Rightarrow g(x) = x^2-2x+1 = (x-1)^2$

$g\circ f(x) = g(f(x)) = (3x-5)^2 = 9x^2+25-30x$

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$f - g = -x + 5$, $f(x) = 2x + 4$

$3f - 2g = 14$, $g(x) = 3x - 1$

$fog(x) = 2(3x-1) + 4 = 6x + 2 \Rightarrow fog(a) = 6a + 2 = 0 \Rightarrow a = -\frac{1}{3}$

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$fog(x) = 4^{\log_2^x} = x^{\log_2^4} = x^2$

$g \circ f(x) = \log_2^{4^x} = \log_2^{2^{2x}} = 2x\log_2^2 = 2x$

$fog(x) \leq g \circ f(x) \Rightarrow x^2 \leq 2x \Rightarrow x^2 - 2x \leq 0$

$* \; g(x) = \log_2^x \Rightarrow x > 0 \;]\; I$

$+ \;|\; 0 \; - \; 2 \;|\; + \Rightarrow 0 \leq x \leq 2 \;]\; II$

اشتراک II, I ← جواب : $(0, 2] \Rightarrow$ اعداد صحیح $= \{1, 2\}$

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$f\left(\frac{x^2 - 2}{x}\right) = f\left(\underbrace{x - \frac{2}{x}}_{t}\right) = \left(x^2 - 4 + \frac{4}{x^2}\right) + x - \frac{2}{x} = \underbrace{\left(x - \frac{2}{x}\right)^2}_{t^2} + \underbrace{\left(x - \frac{2}{x}\right)}_{t}$

$* \left(x - \frac{2}{x}\right)^2 = x^2 + \frac{4}{x^2} - 4$

$\Rightarrow f(t) = t^2 + t \Rightarrow f(x) = x^2 + x$

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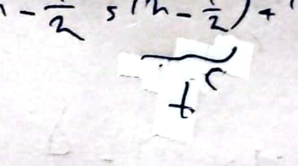
$D_{fog} = \{x \mid x \in D_g , g(x) \in D_f\}$

$\Rightarrow g(x) = 8x - x^2 \Rightarrow D_g = \mathbb{R}$

$f(x) = \log_2^x \Rightarrow x > 0 \Rightarrow 8x - x^2 > 0$ $\quad - \;|\; 0 \; + \; 8 \;|\; - \Rightarrow 0 < x < 8$

$\Rightarrow D_{fog} = (0, 8) = A$



$fog(x) = \log_2^{8x - x^2}$ در بازه $(0, 8)$ برد g برابر است با: $(0, 16]$

$\Rightarrow 0 < g \leq 16 \Rightarrow \log_2^g \leq \log_2^{16} \Rightarrow -\infty < \log_2^g \leq 4$

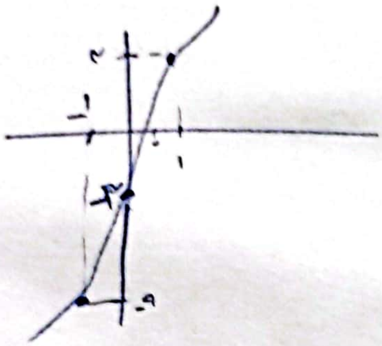
$\Rightarrow R_{fog} = (-\infty, 4] = B$

$\Rightarrow A \cap B = (0, 4] \Rightarrow$ اعداد صحیح $= \{1, 2, 3, 4\}$

$$f(f(x)) = 1 - 3f^2(x) \xrightarrow{f(x)=t} f(t) = 1 - 3t^2 \Rightarrow f(x) = 1 - 3x^2$$ ۹-

$$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 3x - 1 + 2x - 2 + 1$$
$$= x^3 - 3x^2 + 5x - 2$$

$$\Rightarrow h(g(x)) = f(x) \Rightarrow x^3 - 3x^2 + 5x - 2 = 1 - 3x^2 \Rightarrow x^3 + 5x - 3 = 0$$



حال نمودارها را با نقطه رسم کنیم.

با توجه به روند نمودار معادلات که یک ریشه دارد (صعودی)

$$g(f(x)) = ax^3 + 2x \quad \Rightarrow f(x) = 3 \Rightarrow x - \frac{4}{x} = 3$$ ۱۰-

$$g(3) = 8$$

$$\Rightarrow \frac{x^2 - 4}{x} = 3 \Rightarrow x^2 - 3x - 4 = 0 \Rightarrow \begin{cases} x = -1 \\ x = 4 \end{cases}$$

$$\Rightarrow g \circ f(4) = g(3) = 64a + 8 = 8 \Rightarrow \boxed{a = 0}$$

$$g \circ f(-1) = g(3) = -a - 2 = 8 \Rightarrow \boxed{a = -10}$$