به نام خدا

کیوان مکارم نسب — تکلیف شماره ۸

(۱)

$D_{gof} = \{x \in D_f ,\ f(x) \in D_g\} \rightarrow D_f :$ ① $x-1>0 \rightarrow x>1$

② $\log_2^{(x-1)} \geq 0 \rightarrow x-1 \geq 1 \rightarrow x \geq 2$

$\rightarrow D_f = [2, +\infty)$, $g(x) = \sqrt{-(x^2-4x+4)} = g(x) = \sqrt{-(x-2)^2} \rightarrow D_g = \{2\}$

$f(x) = \sqrt{\log_2^{(x-1)}} = 2 \xrightarrow{\text{توان } 2} \log_2^{x-1} = 4 \rightarrow x-1 = 16 \rightarrow x = 17$

$\rightarrow (x=17) \cap [2, +\infty) \Rightarrow D_{gof} = \{17\}$ ✓

(۲)

$fog(x) = f(x^2-3x+8) \Rightarrow fog(x) = 2(x^2-3x+8) - 5 = 2x^2-6x+11$

$gof(x) = g(2x-5) = (2x-5)^2 - 3(2x-5) + 8 = 4x^2-20x+25-6x+15+8$

$= gof(x) = 4x^2-26x+48 \longrightarrow 4x^2-26x+48 = 2x^2-6x+11$

$\rightarrow 2x^2-20x+37 = 0 \xrightarrow[\text{ریشه‌های معادله}]{\alpha, \beta} \alpha^2+\beta^2 = S^2-2P = 100-37 = \boxed{63}$ ✓

$(S = \frac{20}{2} = 10 ,\ P = \frac{37}{2})$

(۳)

$g(2x) = 5x^2+11 \longrightarrow 2x = t \rightarrow x = \frac{t}{2} \rightarrow g(t) = 5(\frac{t}{2})^2 + 11 = 5(\frac{t^2}{4}) + 11$

$\Rightarrow g(x) = \frac{5x^2}{4} + 11 \longrightarrow g(x-7) = \frac{5(x-7)^2}{4} + 11 \rightarrow g(x-7) = \frac{5(x^2-14x+49)}{4} + 11$

$g(x-7) = \frac{5x^2 - 70x + 245 + 44}{4} = \frac{5x^2}{4} - \frac{35}{2}x + \frac{289}{4}$

راس سهمی: $x = \frac{-b}{2a} = \frac{\frac{35}{2}}{\frac{5}{2}} = 7$ ✓ $\longrightarrow y_{\text{راس}} = \frac{5(49)}{4} - \frac{35}{2}(7) + \frac{289}{4} = \frac{245}{4} - \frac{490}{4} + \frac{289}{4}$

$= \frac{44}{4} = 11$ کمترین مقدار ✓

(۴)

$f(g(x)) = 3x^2-6x-1 \longrightarrow 2g(x) - 4 = 3x^2-6x-1$

$\rightarrow 2g(x) = 3x^2-6x+3 \longrightarrow g(x) = \frac{3}{2}x^2 - 3x + \frac{3}{2} \rightarrow g(f(x)) = \frac{3}{2}(3x-4)^2 - 3(3x-4) + \frac{3}{2}$

$\Rightarrow \frac{3}{2}(9x^2-24x+16) - 9x + 12 + \frac{3}{2} = \frac{27}{2}x^2 - 36x + 24 - 9x + 12 + \frac{3}{2}$

$gof(x) = \frac{27}{2}x^2 - 45x + \frac{75}{2}$ ✓

(5)

$F-g(x) = -x+5 \longrightarrow F(x) - g(x) = -x+5$ ①

طبق فرض, $3f(x) - 14 = 2g(x) \longrightarrow g(x) = \frac{3}{2}f(x) - 7 \xrightarrow{①} f(x) - \frac{3}{2}f(x) + 7 = -x+5$

$\rightarrow f(x) = 2x+4 \;,\; g(x) = 3x-1 \longrightarrow f\circ g(x) = 2(3x-1)+4$

$= 6x - 2 + 4 \Rightarrow f\circ g(x) = 6x+2 \xrightarrow{\text{شیب}} \frac{2}{a} = 6 \rightarrow \boxed{a = \frac{-1}{3}}$ ✓

(6)

$f\circ g(x) \le g\circ f(x) \longrightarrow f\circ g(x) = 9^{(\log_3 x)} \xrightarrow[\text{لگاریتم}]{\text{خاصیت}} x^{\log_3 9} = x^2$

$g\circ f(x) = \log_3 9^x = x\log_3 9 = 2x$

$\rightarrow x^2 \le 2x \longrightarrow x^2 - 2x \le 0 \longrightarrow (x)(x-2) \le 0$ $0 \quad 2$: $+ \;|\; - \;|\; +$

$\rightarrow (0,2]$ ✓ $\xrightarrow{x\in Z} \{1,2\}$ ✓

$x \ne 0$

(7)

$f\left(x - \frac{2}{x}\right) = x^2 + \frac{4}{x^2} + x - \frac{2}{x} - 4 \xrightarrow{x-\frac{2}{x}=t} f(t) = t^2 + 8 + t - 4$

$\rightarrow \boxed{f(t) = t^2+t+4} \longrightarrow x - \frac{2}{x} = t \longrightarrow \frac{x^2-2}{x} = t \rightarrow x^2 - 2 = tx$

$x^2 - tx - 2 = 0 \rightarrow x = \frac{t \pm \sqrt{t^2+8}}{2} \longrightarrow$

(8)

$f\circ g(x) = \log_2^{-x^2+8x} \xrightarrow{①} D_f = -x^2+8x > 0 \longrightarrow x^2 - 8x < 0$

$\rightarrow x(x-8) < 0$ $0 \quad 8$: $+ \;|\; - \;|\; +$ $\longrightarrow D_f = A = (0,8)$

$R_F \Rightarrow$ راس $x = \frac{-b}{2a} = \frac{-8}{-2} = 4 \rightarrow$ راس $y = -16+32 = 16 \rightarrow (-\infty,16]$ $\log_2(-\infty,4] = B$

$A \cap B \rightarrow$ $0 \quad 4 \quad 8$ $\rightarrow (0,4]$ ✓

(9)

$h(g(x)) = (x-1)^3 + 2(x-1) + 1 = x^3 - 3x^2 + 2x - 2$

$F(f(x)) = 1 - 3f^2(x) \xrightarrow{f(x)=t} f(t) = 1 - 3t^2 \longrightarrow f(x) = 1 - 3x^2$

$\rightarrow x^3 - \cancel{3x^2} + 2x - 2 = 1 - \cancel{3x^2} \longrightarrow x^3 + 2x - 3 = 0 \xrightarrow[\text{بر } x-1 \text{ بخش پذیر}]{\text{جمع ضرایب } 0}$

$x^3 + 2x - 3 = (x-1)\underbrace{(x^2+x+3)}_{\text{همواره مثبت}} = 0 \longrightarrow \boxed{x=1}$

(10)

$f(x) = x - \frac{4}{x} \;,\; g\left(x - \frac{4}{x}\right) = ax^3 + 2x \xrightarrow{g(3)=8} x - \frac{4}{x} = 3 \xrightarrow{\times x}$

$x^2 - 3x - 4 = 0 \longrightarrow x = -1 \;,\; x = 4 \xrightarrow{x=-1} -a - 2 = 8 \longrightarrow \boxed{a = -10}$

$x = 4 \longrightarrow 64a + 8 = 0 \longrightarrow \boxed{a = -\frac{1}{8}}$