① $f(2x+1)+f(x+2)=3x+f(3)$ تابع R $y=\sqrt{f(x)-x^2}$

تابع خطی $\longrightarrow f(x)=ax+b \rightarrow f(2x+1)+f(x+2)=3x+f(3)$

$2ax+a+b+ax+2a+b=3x+3a+b \rightarrow 3ax+b=3x \rightarrow b=0$

$3ax=3x \longrightarrow a=1 \rightarrow f(x)=x$ $\quad y=\sqrt{x-x^2}$ $\quad R_{کل}=[0,\frac{1}{2}]$

$R_{زیر رادیکال}=(-\infty,\frac{1}{4}]$

---

② $f(x)=\frac{x^3-4x}{x+2}$ $D_f=R-\{a,b\}$ $\quad R_f=[c,+\infty)-\{8\}$

$x^2-2x \rightarrow$ از min تا $+\infty$

8

$\frac{x(x^2-4)}{x+2}=\frac{x(x-2)\cancel{(x+2)}}{\cancel{x+2}} \xrightarrow{x\neq -2} R=[-\frac{1}{2},+\infty)$

$R=8 \longrightarrow D=\{4,-2\}$ $\quad D_f=R-\{4,-2\}$ $\quad f(\frac{2}{-1})=f(-2)$

$x^2-2x \rightarrow 4+4=8$

$x^2-2x=8 \rightarrow x^2-2x-8=0 \rightarrow (x-4)(x+2)=0 \rightarrow 4,-2$

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③ $y=\sqrt[3]{x^3+ax^2+bx-8}$, $[a,b]$ تابع خطی

$y=\sqrt{مکعب کامل} \longrightarrow \sqrt[3]{(x-2)^3}=\sqrt[3]{x^3-6x^2+12x-8}$ $\quad [-8,10]$

$D_f=[-6,12] \longrightarrow R_f=[-8,10]$

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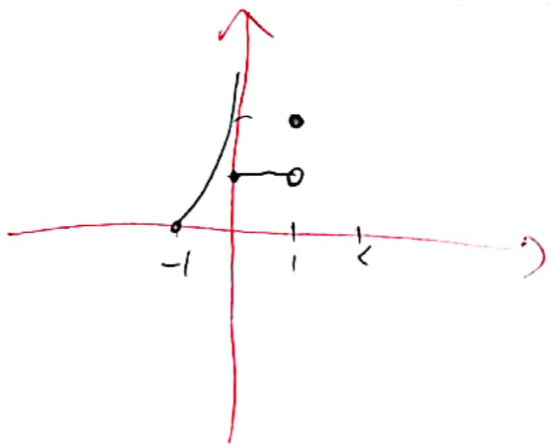
④ $y_1=(\frac{x+1}{1-x})^{\frac{1}{5}} \longrightarrow (\frac{x+1}{1-x})\geq 0$ $\quad a+b=?$ $\quad \rightarrow -1\leq x<1$

$y_2=\frac{ax^2-1}{x^2+b}$ $R_{y_2}\longrightarrow[-\frac{1}{b},a) \longrightarrow b=1, a=1$ $\quad a+b=2$

$a+b=2$

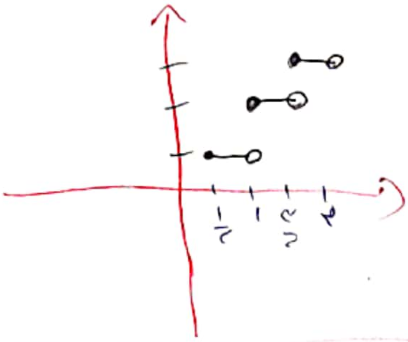
---

⑤ $\frac{[x]}{x}+\frac{|x|}{x}$ $\quad [-1,1]$



۶)

$y=[x]+[x+\frac{1}{2}]$ بازه $[-\frac{1}{2},2)$

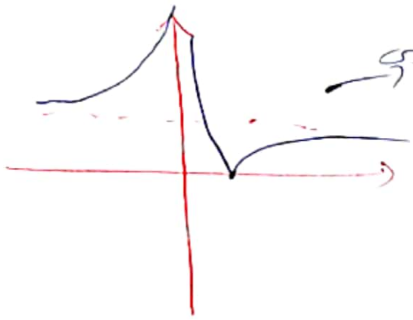


۷)

$\left|\frac{x-1}{ax+2}\right|$

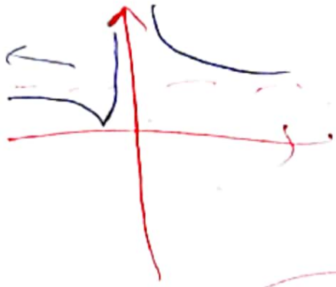
$y=1$

$(b,+\infty)$



محوری

نزولی



$a=1$

$b=-2$

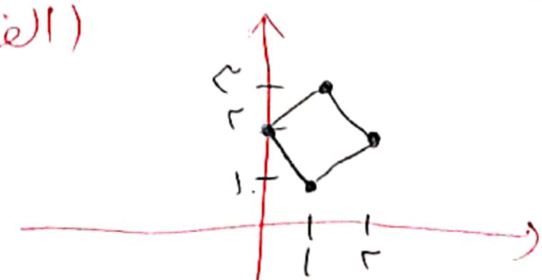
$a-b=3$

$a-b=3$

۸)

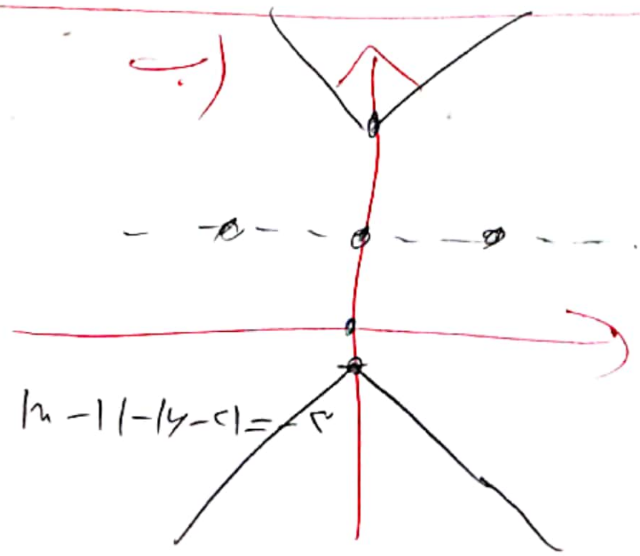
۹)

الف)



$|x-1|+|y-2|=1$

ب)



$|x-1|-|y-2|=-2$

$y = 2x + |ax+b| \qquad R_f = \mathbb{R}$ $\qquad -2 \leq a \leq 2$ $\qquad 10$

$ax+b \geq 0 \longrightarrow 2x+ax+b = y \longrightarrow y=(2+a)x+b$

$ax+b \leq 0 \longrightarrow 2x-ax-b = y \longrightarrow y=(2-a)x-b$

$\longrightarrow f(-\frac{b}{a}) = f(-\frac{b}{a})$

$(2+a)(2-a) \geq 0 \qquad -2 \leq a \leq 2 \qquad (2+a)(-\frac{b}{a})+b = (2-a)(-\frac{b}{a})-b$