

الف) $y = \left(x - 2\left[\frac{x}{2}\right]\right)^2 = \left(2\left(\frac{x}{2} - \left[\frac{x}{2}\right]\right)\right)^2 \rightarrow (0 \leq x < 2) \rightarrow 0 \leq y < 4$
 $R_f = [0, 4)$

ب) $-\sin x + \log y = -f \Rightarrow \log y = f - \sin x \Rightarrow 3 \leq \log y \leq 5 \rightarrow 10^3 \leq y \leq 10^5$
 $\rightarrow R_f = [10^3, 10^5]$

$f(x) = \sqrt{\frac{x^2 - 4}{x^2 - 9}} \rightarrow g = \frac{x^2 - 4}{x^2 - 9} = \frac{0 - 4}{0 - 9} = \frac{4}{9}$ (نقطه) $\rightarrow$ $\frac{4}{9}$ (نقطه)
 $+ \infty \Rightarrow \frac{4}{9} \rightarrow R_g = (-\infty, \frac{4}{9}] \cup (1, +\infty) \rightarrow R_f = \sqrt{(-\infty, \frac{4}{9}] \cup (1, +\infty)}$
 $= \left[0, \frac{2}{3}\right] \cup (1, +\infty) \rightarrow a+b+c = \frac{5}{3}$

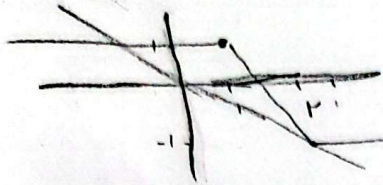
الف) $y = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \rightarrow x + \frac{1}{x} \geq 0 \rightarrow x > 0 \rightarrow a + \frac{1}{a} \geq 0 \rightarrow R_f = [2, +\infty)$
 $\rightarrow (x + \frac{1}{x}) + 2\sqrt{x + \frac{1}{x}} \Rightarrow R_f = [x + \frac{1}{x} + 2, +\infty)$
 $g = \left(\sqrt{x + \frac{1}{x}} + 1\right)^2 - 1 \rightarrow \min \left(x + \frac{1}{x}\right) = 2 \rightarrow (2 + 1)^2 - 1 = 4$
 $R_f = [4, +\infty)$
 ب) $y = (3 - \sin x)(1 + \sin x) = 3 - \sin^2 x + 2\sin x + 3$
 $\rightarrow R_f = [0, 4]$
 $\sin x = 1 \Rightarrow 4$
 $\sin x = -1 \Rightarrow 0$
 $\sin x = \frac{1}{2} \Rightarrow 4$

$f(n) = -\frac{1}{n}n + 10 \rightarrow P_f = \{1, 2, 3, 4, 5, \dots, 9\}$
 $\rightarrow$ مجموع اعداد 1 تا 9 : $-\frac{1}{9}(1 + 2 + \dots + 9) + 9 \times 10 = -\frac{1}{9} \times \frac{9 \times 10}{2} + 90 = 85$

الف) $f(x) = \sqrt{ax^2 + bx + c} \rightarrow P_f = [2, +\infty)$
 $\sqrt{bx + c} \rightarrow bx + c \geq 0 \rightarrow 2bx + c \geq 0$
 $f(3) = 1 \rightarrow \sqrt{9b + c} = 1 \rightarrow 9b + c = 1 \rightarrow b = 1, c = -8$
 $g(x) = \sqrt{bx^2 + ax - 2c} = \sqrt{x^2 + 4} \rightarrow R_g = \sqrt{[4, +\infty)} = [2, +\infty)$

$$|n-2| - |n-1|, u \in \mathbb{R}$$

محل دارنده بار

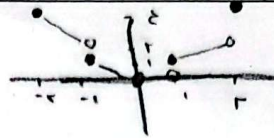


$$\text{خطای محاسبه} \rightarrow 2u = -1 \rightarrow u = -\frac{1}{2}$$

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$$y = \alpha[n]$$

$$\begin{aligned} -1 \leq \alpha \leq -1 &\rightarrow y = -2\alpha \\ -1 \leq \alpha < 0 &\rightarrow y = -\alpha \\ 0 \leq \alpha < 1 &\rightarrow y = 0 \\ 1 \leq \alpha < 2 &\rightarrow y = \alpha \\ \alpha \geq 2 &\rightarrow y = 2\alpha \end{aligned}$$

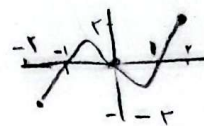


$$R_f, [0, \infty], \{2\}$$

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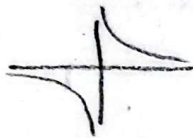
$$y = \alpha|n| - \alpha$$

$$\begin{aligned} -1 \leq \alpha \leq 1 &\rightarrow y = \alpha - \alpha \\ -1 \leq \alpha < 0 &\rightarrow y = -\alpha - \alpha \end{aligned}$$



$$R_f, [-1, 1]$$

$$y = \frac{1}{n} + \frac{1}{n+1} + \frac{1}{n^2 + \alpha} = \frac{n+1}{n^2 + n} + \frac{1}{n^2 + \alpha} = \frac{2(n+1)}{n(n+1)} = \frac{2}{n}$$



$$P_f, R, \{0, -1\} \rightarrow R_f, R, \{0, -1\}$$

$$f(-1) = -1$$

$$\text{atb} = -1$$

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$$f(n) = \frac{n - \sqrt{n+1}}{n - \sqrt{n+1} + 1} = \frac{(\sqrt{n+1} - 1)(\sqrt{n+1} - 1)}{(\sqrt{n+1} - 1)(\sqrt{n+1} - 1)} = \frac{\sqrt{n+1} - 1}{\sqrt{n+1} - 1} \rightarrow \sqrt{n+1} = +\infty = 1$$

$$R_f, (1, \infty]$$

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$$f(n) = \frac{n^2 + n^2 - n - 1}{n^2 - 1} = \frac{n^2(n+1) - (n+1)}{n^2 - 1} = \frac{n+1(n^2-1)}{n^2-1} = n+1$$

$$n \geq 1 \rightarrow y = n$$

$$n = -1 \rightarrow y = 1$$

$$\rightarrow \text{atb} = \{1, \infty\}$$

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