

$$\frac{x^2 + x + 2}{x-1} = y \rightarrow yx - y = x^2 + x + 2 \rightarrow x^2 + (1-y)x + 2+y = 0$$

$$\Delta \geq 0 \Rightarrow x = \frac{y-1 \pm \sqrt{y^2 - 4y - 7}}{2} \rightarrow y^2 - 4y - 7 > 0 \rightarrow y \in \mathbb{R} - [-1, 7]$$

۱- د و ۷ در برتا بوسند!

$y = \mathbb{R} - (-1, 7)$

الف) $2x^2 - 5x + 1 \rightarrow x_{min} = \frac{5}{4}$ } $y = [-\frac{17}{4}, +\infty)$

ب) $\sqrt{-x^2 + 4x - 5} \rightarrow y_{max} = \sqrt{4} = 2 \Rightarrow y \in [0, 2]$

ج) $y \in [0, +\infty)$

د) $y \in \mathbb{R}$

الف) $y = \frac{x^2 + 1}{x^2 - 4} \rightarrow y \in \mathbb{R} - \{\frac{1}{4}\}$

ب) $y = \sqrt{\frac{x+1}{x-2}} \rightarrow y \in [0, +\infty) - \{\frac{1}{2}\}$

ج) $y = \frac{x^2 + 3 + 1}{\sqrt{x^2 + 3}} = \sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \rightarrow y \in [\frac{4}{\sqrt{3}}, +\infty)$

د) $x^2 + 3 + \frac{1}{x^2 + 3} - 4 \rightarrow y \in [\frac{1}{4}, +\infty)$

$$yx^2 - y = x^2 + 2 \rightarrow (y-1)x^2 + 2 + y = 0$$

$$x = \frac{-2 \pm \sqrt{4 - 4(y-1)(2+y)}}{2(y-1)} \rightarrow y \in (-\infty, -4] \cup (3, +\infty)$$

خارج منقاد منفرد

$x^2 = +\infty \rightarrow 3$

$x^2 = 0 \rightarrow -4$

$(-\infty, -4] \cup (3, +\infty)$

$$y \sin x + 2y = 2 \sin x - 1$$

$$(y-2) \sin x - 1 - 2y = 0$$

$$\sin x = \frac{2y+1}{y-2} \rightarrow -1 \leq \frac{2y+1}{y-2} \leq 1$$

خارج منفرد شد

$y \in [-\frac{3}{2}, \frac{1}{2}]$

$$f(x) = \frac{(x - \frac{1}{r})^r}{x^r - x + 1} \rightarrow D_f \in \mathbb{R}$$

$\hookrightarrow \Delta r.$

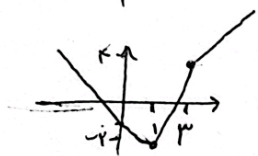
$$\hookrightarrow y = \frac{x^r - x + \frac{1}{r}}{x^r - x + 1} \rightarrow yx^r - yx + y = x^r - x + \frac{1}{r}$$

$$\hookrightarrow (y-1)x^r - (y-1)x + y - \frac{1}{r} = 0 \rightarrow x = \frac{y-1 \pm \sqrt{-ry^r + ry}}{r(y-1)}$$

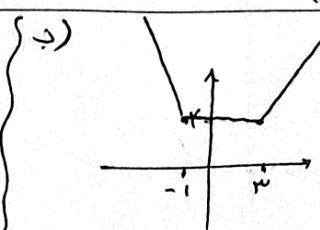
$$R_f \in [0, 1]$$

$$-ry^r + ry \geq 0$$

الف) $\begin{array}{c|c|c} -x-1 & rx-0 & x+1 \\ \hline -r & r & \end{array}$

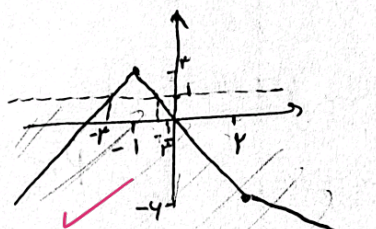


$$\rightarrow R_f \in [-r, +\infty)$$

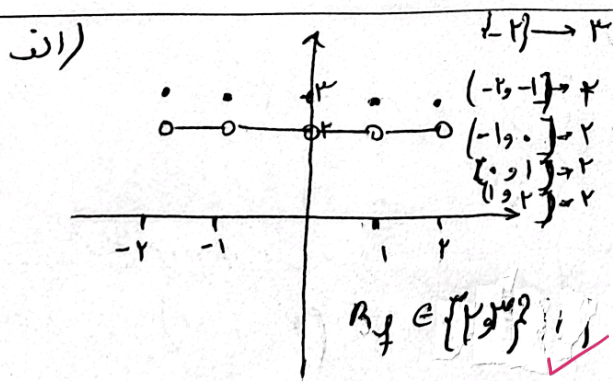


$$R_f \in [r, +\infty)$$

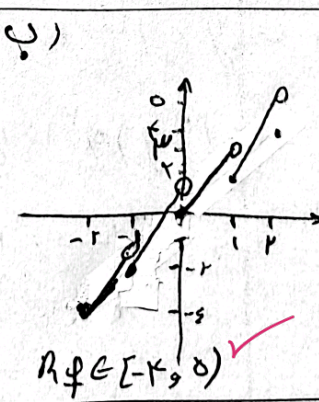
$\begin{array}{c|c|c} -1 & r & \\ \hline x+\varepsilon & -rx & -x-\varepsilon \\ \hline r & -r & \end{array}$



$$x \in \mathbb{R} - (-r, -\frac{1}{r})$$



$$R_f \in [r, +\infty)$$



$$R_f \in [-r, 0)$$

$\begin{array}{l} [-r, -1] \rightarrow rx+r \\ [-1, 0] \rightarrow rx+1 \\ [0, 1] \rightarrow rx \\ [1, r] \rightarrow rx-1 \\ \{r\} \rightarrow rx-r \end{array}$

الف) $\sin \left\{ \begin{array}{c} 1 \\ \frac{1}{r} \\ \frac{1}{r} \\ \frac{1}{r} \end{array} \right\} R_f \in [1, r]$
 ب) $\sin^2 x \left\{ \begin{array}{c} 1 \\ \frac{1}{r} \\ \frac{1}{r} \\ \frac{1}{r} \end{array} \right\} R_f \in [1, r]$

ب) $\sin^2 x \left\{ \begin{array}{c} 1 \\ \frac{1}{r} \\ \frac{1}{r} \\ \frac{1}{r} \end{array} \right\} R_f \in [1, r]$