(۱

$$\lim_{x\to 0^+} \frac{f(x)}{x} = 0 \to f'_+(0) = 0$$

$$\lim_{x\to 0^-} \frac{f'(x)}{x} = 2 \to f''_-(0) = 2$$

$$f(0) = 1+b = 0 \to b = -1$$

$$f'(x) = 3\cos^2 2x\,(-2\sin 2x) + 2ax$$

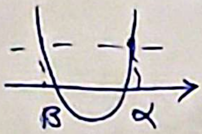
$$f''(x) = 6\cos 2x\,(-2\sin 2x)(-2\sin 2x) + 3\cos^2 2x\,(-4\cos 2x) + 2a \Big|_{x=0} \to -12 + 2a = 2$$

$$a = 7$$

$$7 + (-1) = 6$$

(۲

$$y' = 2x$$



$$2\alpha \times 2\beta = -1 \to \alpha\beta = -\frac{1}{4}$$

$$\alpha = -\beta \qquad -\alpha^2 = -\frac{1}{4} \to \alpha = \frac{1}{2}, \quad \beta = -\frac{1}{2}$$

$$\left(\frac{1}{4} - 1\right)2 = -\frac{3}{2}$$

(۳

$$m = \frac{6+12}{1.5+1.5} = \frac{18}{3} = 6 \to d: 6(x-1.5) = y-6 \to y = 6x-9$$

$$f(x) = a \xrightarrow{\Delta = 0} \frac{a}{2x-1} = 6x - 9 \to a = 12x^2 - 24x + 9 \to 12x^2 - 24x + 9 - a$$

$$12^2 - 12(9-a) = 0 \to a = -3$$

$$f'(x) = \frac{-3}{2x-1}\Big|_{x=5} \to \frac{-3}{9} = -\frac{1}{3}$$

(۴

$$y' = \frac{1-a^2}{(ax+1)^2}\Big|_{x=1} \to \frac{1-a^2}{(a+1)^2} = 2 \quad a = -1 \ \times, \quad a = -\frac{1}{3}\ \checkmark$$

$$\to y = \frac{x-1}{1-x} = -1 \to \text{تابع ثابت } \times$$

$$\to y = \frac{x - \frac{1}{3}}{-\frac{1}{3}x + 1}\Big|_{x=1} \to y = 1$$

$$1 = 2(1) + b \to b = -1$$

$$-\frac{1}{3} + 1 = \frac{2}{3}$$

۵)

$f=g \to \frac{3}{2}\sin x = \sin x + \frac{1}{2}\cos x \to \frac{1}{2}\sin x = \frac{1}{2}\cos x \xrightarrow{x\in[0,\pi]} x=\frac{\pi}{4}$

$f'(x) = \cos x - \frac{1}{2}\sin x \Big|_{x=\frac{\pi}{4}} \to f'(\frac{\pi}{4}) = \frac{\sqrt{2}}{4}$

$d: \frac{\sqrt{2}}{4}(x-\frac{\pi}{4}) = y - \frac{3\sqrt{2}}{4} \xrightarrow[y=0]{\text{محل برخورد}} \frac{\sqrt{2}}{4}(x-\frac{\pi}{4}) = -\frac{3\sqrt{2}}{4} \to x = \frac{\pi}{4} - 3$

$f(\frac{\pi}{4}) = \frac{3\sqrt{2}}{4}$

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۶)

$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$

$6(x-2)(x+1) = 0$

x		-1		2	
f'	$+$		$-$		$+$
f	$\nearrow$	max	$\searrow$	min	$\nearrow$

$f(2) = 16 - 12 - 24 + 1 = -19$

$f(-1) = -2 - 3 + 12 + 1 = 8$

$m_{AB} = \frac{8+19}{-1-2} = -9$

$f'(x) = -9 \to 6x^2 - 6x - 12 = -9 \to 6x^2 - 6x - 3 = 3(2x^2 - 2x - 1)$

$\Delta > 0 \to$ ۲ تا ریشه $\to$ ۲ تا مماس با شیب -9

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۷)

$x = \frac{-b}{3a} = \frac{-k-1}{3k}$

	-1		0	
$-$		$+$		$-$

$k<0 \to k<-1$

$x_0 \neq 0$

$f(x_0) > 0$

$y = x^2(kx + k + 1)\Big|_{x=\frac{-k-1}{3k}}$ $\left(\frac{k+1}{3k}\right)^2\left(\frac{-k-1+3k+3}{3}\right)$

$\frac{2k+2}{3} > 0 \to k > -1$

همجوار منفی ۲

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۸)

$y' = 3x^2 + 2ax + b$

$y'' = 6x + 2a\Big|_{x=-1} = 0 \to -6 + 2a = 0 \to a = 3 \to \frac{a}{b} = 0/6$

$y(-1) = -4 \to \cancel{-1} + \cancel{3} - b - \cancel{1} = -4 \to b = 5$

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۹)

$f(0) = 4 \to c = 4$

$f'(0) = 0 \to 3x^2 + 2ax + b\Big|_{x=0} = 0 \to b = 0$

$f'(x) = x(3x + 2a) \to$ min نسبی $= \frac{-2a}{3} \xrightarrow{a=-3} \frac{-2(-3)}{3} = 2$

$f(-\frac{2a}{3}) = 0 \quad \frac{-8a^3}{27} + \frac{4a^3}{9} + 4 = 0$

$\frac{-8a^3 + 12a^3}{27} = -4 \to a = -3$

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۱۰)

$f'(x) = 4x^3 - 12x = 4x(x^2 - 3)$

$f''(x) = 12x^2 - 12 = 12(x-1)(x+1)$

x		$-\sqrt{3}$		0		$\sqrt{3}$	
f'	$-$		$+$		$-$		$+$
f	$\searrow$		$\nearrow$		$\searrow$		$\nearrow$

$x = \pm\sqrt{3} \to$ min نسبی

x		-1		1	
f''	$+$		$-$		$+$

$f(x) = (x^2-1)(x^2-5)$

$f(\pm\sqrt{3}) = (3-1)(3-5) = -4 \qquad m_{AB} = \frac{-4-(-4)}{2\sqrt{3}} = 0$

$f(\pm 1) = 0 \qquad m_{CD} = 0$

دو خط موازی اند

نوید امینی ـ کلاس ۱۲

CamScanner