

« بینا خدا »

کلیف شماره ۲۴

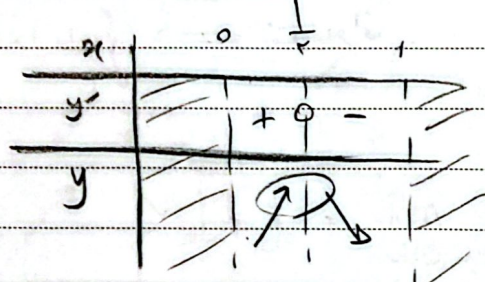
در کیوان (مطابق نسبت) <<

$$f(x) = \begin{cases} \sqrt{x-x^2} & 0 < x < 1 \\ \sqrt{x+x^2} & x < -1 \end{cases} \rightarrow f'(x) = \begin{cases} \frac{1-2x}{2\sqrt{x-x^2}} & 0 < x < 1 \\ \frac{2x+1}{2\sqrt{x+x^2}} & x < -1 \end{cases} \quad (1)$$

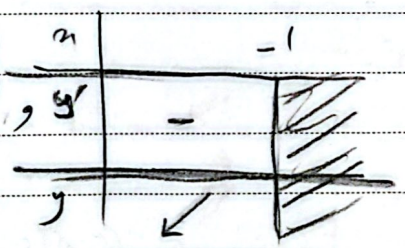
نقاط بحرانی $\Rightarrow x^2+x=0 \rightarrow x=0, x=-1$ و $x-x^2=0 \rightarrow x=0, x=1$

$K = \varepsilon$

نقاط \rightarrow Max
Min



$M < 1$



$\rightarrow K, M, n = \varepsilon + 1 + 0 = \boxed{1+\varepsilon}$

$n=0$

$(D) f = [0, +\infty) \rightarrow (P) a-2x > 0 \rightarrow a > 2x \rightarrow x < \frac{a}{2} \rightarrow f = [0, \frac{a}{2}]$

$$f'(x) = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt{a-2x}} = 0 \rightarrow \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{a-2x}} \rightarrow \sqrt{x} = \sqrt{a-2x}$$

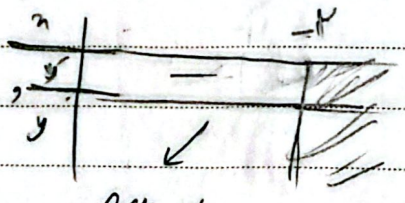
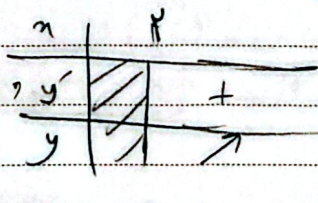
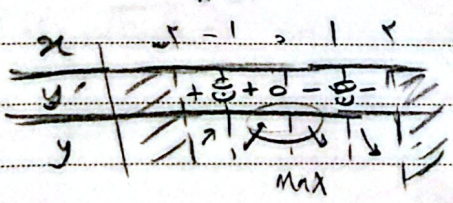
$\varepsilon x = a-2x \rightarrow x = \frac{a}{3} \rightarrow$ نقاط بحرانی $= \{0, \frac{a}{3}, \frac{a}{2}\}$

$f(0) = \sqrt{a}$
 $f(\frac{a}{3}) = \sqrt{\frac{a}{3}} \rightarrow f(\frac{a}{3}) = \sqrt{\frac{a}{3}} + \sqrt{\frac{2a}{3}} = \sqrt{3} \sqrt{\frac{a}{3}}$

$\rightarrow \text{Max} = \sqrt{3} \sqrt{\frac{a}{3}}, \text{Min} = \sqrt{\frac{a}{3}} \rightarrow \sqrt{\frac{a}{3}} \times \sqrt{3} \sqrt{\frac{a}{3}} = \boxed{a}$

$[E] = \varepsilon$

$$f(x) = \begin{cases} \frac{-x\varepsilon + \varepsilon x^2}{x^2-1} & -r < x < r \\ \frac{x\varepsilon - \varepsilon x^2}{x^2-1} & x < -r \text{ or } x > r \end{cases} \rightarrow f'(x) = \begin{cases} \frac{-\varepsilon x^2}{(x^2-1)^2} & -r < x < r \\ \frac{\varepsilon x^2}{(x^2-1)^2} & x < -r \text{ or } x > r \end{cases}$$



$-r < x < r \rightarrow x=0 \rightarrow \text{Min}$

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ریشه‌های تکرارمقلوب افزا می‌شوند!

مقدار ε در نقطه دارد

$f(x) = ax^2 + bx^2 + cx + d \rightarrow f(0) = 0 \rightarrow d = 0$ (E)

$f'(x) = 2ax^2 + 2bx + c \rightarrow f'(0) = 0 \rightarrow c = 0$

① $f(1) = 1 \rightarrow \begin{cases} a+b=1 \\ 3a+2b=2 \end{cases} \rightarrow \begin{cases} a=-2 \\ b=3 \end{cases} \rightarrow ab = -6$ ✓
 ② $f'(1) = 0 \rightarrow \begin{cases} 3a+2b=0 \\ 3a+2b=0 \end{cases}$

$f(x) = \begin{cases} x^2 - x^2 & -\sqrt{x} \leq x \leq \sqrt{x} \\ x^2 - \sqrt{x} & x > \sqrt{x} \text{ or } x < -\sqrt{x} \end{cases} \rightarrow f'(x) = \begin{cases} 2x - 2x & -\sqrt{x} \leq x \leq \sqrt{x} \\ 2x - \frac{1}{2\sqrt{x}} & x > \sqrt{x} \text{ or } x < -\sqrt{x} \end{cases}$ (E)

$2x - 2x = 0 \rightarrow x = \pm 1$ ✓ $x \in D_f \Rightarrow \{-1, 1, \sqrt{x}\}$

$2x - \frac{1}{2\sqrt{x}} = 0 \rightarrow x = \pm 1$ ✗

$f(-1) = -1 + 1 = 0$
 $f(1) = 1 - 1 = 0$
 $f(\sqrt{x}) = x - \sqrt{x} = 0$
 Min = $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$ ✓

$f(-\frac{1}{\sqrt{x}}) = -1 + \frac{1}{\sqrt{x}}$ $f(-1) = 1 \rightarrow 1 + 3a + b = 1 \rightarrow b = -3a$ (E)

$f(x) = \begin{cases} x^2 + 3ax^2 + b & x > 0 \\ -x^2 + 3ax^2 + b & x < 0 \end{cases} \rightarrow f'(x) = \begin{cases} 2x + 6ax & x > 0 \\ -2x + 6ax & x < 0 \end{cases}$

$f'(-1) = 0 \rightarrow -2 - 6a = 0 \rightarrow 6a = -2 \rightarrow a = -\frac{1}{3}$
 $b = -3(-\frac{1}{3}) = 1$
 $\frac{b}{a} = \frac{1}{-\frac{1}{3}} = -3$ ✓

$y = \frac{1}{3}x^2 + x + \frac{1}{3} \rightarrow x = \frac{-b}{2a} = \frac{-1}{\frac{2}{3}} = -\frac{3}{2}$ (E)

$y_{max} = y_{min} = \frac{1}{3}(\frac{1}{9}) - \frac{1}{3} + \frac{1}{3} = \frac{1}{9} - \frac{1}{9} + \frac{1}{3} = \frac{1}{3} \rightarrow \text{Ext} = \begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}$

$\lim_{x \rightarrow \infty} \frac{ax + 3}{(a+1)x + (a-1)} = \frac{a}{a+1} = \frac{1}{2}$ $\rightarrow a = 2$

$(a+1)x + (a-1) < 0 \rightarrow x < \frac{1-a}{a+1} = -\frac{1}{2}$

$y = \frac{2x+3}{x+1} = 0 \rightarrow x = -\frac{3}{2}$ ✓

$$f(x) = \pm \frac{x^2(x^2-2)}{x^2-1} \rightarrow f'(x) = \pm \frac{(4x^3-2)(x^2-1) - (x^4-2x^2)2x}{(x^2-1)^2} = 0 \quad -3$$

$$\pm (4x^3 - 4x^2 + 2x) = 0 \rightarrow x = 0$$

$$\hookrightarrow x^4 - 2x^2 + 2 = 0 \quad (\text{ریشه ندارد})$$

حدا ۲، ۲ - ریشه‌های تکراری و تعدادی ریشه‌های غیر تکراری است پس 3 نقطه‌ای همزی دارد!