

$$\frac{f(x) - f(1)}{x - 1} = \frac{1 - \frac{a}{x} - 1 + a}{x - 1} = \frac{a}{x}$$

$$f(x) = \frac{a}{x^2} \quad \frac{a}{x^2} = \frac{a}{x^2} \xrightarrow{a \neq 0} x^2 = \frac{a}{a} \rightarrow x = \pm \sqrt{a}$$

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$$y = x \quad x < 0$$

$$\text{Max } x^2 - 2x + 1 \text{ (a) } = x$$

$$\text{Max } x^2 - 2x + 1 = 0 \rightarrow \Delta = 0 \rightarrow 4 - 4(1)(1) = 0 \rightarrow x = \frac{2 \pm \sqrt{0}}{2} = 1 \rightarrow a = \pm \frac{1}{x}$$

$$a = \frac{1}{x} \rightarrow x^2 - 2x + 1 = 0 \rightarrow (x-1)^2 = 0 \rightarrow x = 1 \rightarrow x = 1$$

$$a = -\frac{1}{x} \rightarrow -x^2 - 2x - 1 = 0 \rightarrow -(x+1)^2 = 0 \rightarrow x = -1 \rightarrow a = -\frac{1}{x}$$

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$$f' = 2x - 2$$

x	-۲	۲
f	+	-
f	↗	↘
	max	min

$$f(x) = (x^2 - 1)^2 + 2 = -1$$

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$$f' = 2x^2 + 2ax - 2b$$

$$x = 0 \rightarrow f' = 0 \rightarrow b = 0$$

$$x = -1 \rightarrow f' = 0 \rightarrow 1 - 2a = 0 \rightarrow a = \frac{1}{2}$$

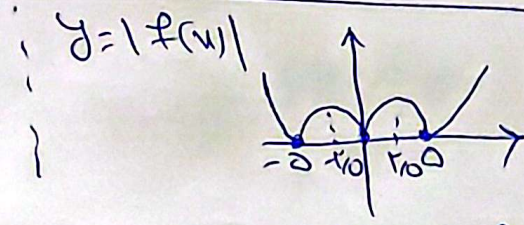
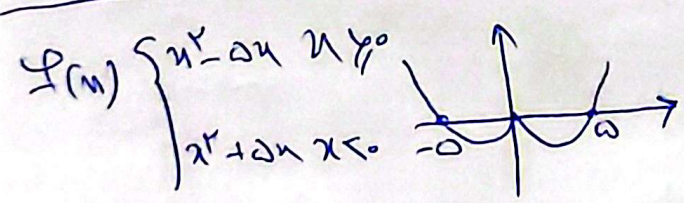
$$f = x^2 + 2x^2 - 2$$

$$f(0) = -2 \rightarrow (0, -2)$$

$$f(-1) = 0 \rightarrow (-1, 0) \rightarrow a = \sqrt{1+2} = \sqrt{3}$$

(۲, ۳)

-۴



$$x = -2 \rightarrow \text{min} \rightarrow m = 2$$

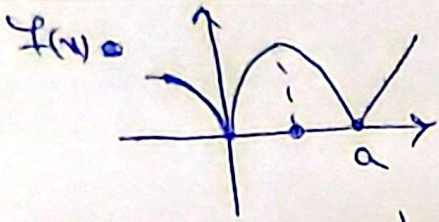
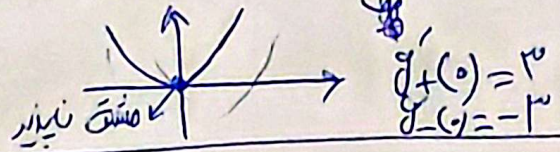
$$x = \pm 2 \rightarrow \text{max} \rightarrow m = 4$$

$$\frac{n}{m} = \frac{2}{4} = \frac{1}{2}$$

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$$|f(x)| = |x(x+3)| = |x|(x+3) \quad \begin{cases} x \geq 0 \rightarrow x^2 + 3x \\ x < 0 \rightarrow x^2 - 3x \end{cases} \quad -6$$

نقطه بحرانی از نوع مسطح ناپذیر



$$f(x) \rightarrow f(u) = a x^{\frac{1}{p}} - x^{\frac{q}{p}} \rightarrow f(u) = \frac{1}{p} a u^{-\frac{1}{p}} - \frac{q}{p} x^{\frac{q}{p}}$$

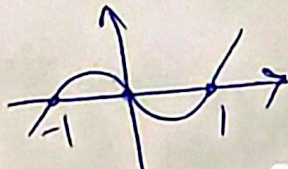
$$f(u) = \frac{1}{p} u^{-\frac{1}{p}} (pa - qu) = 0$$

$$f\left(\frac{pa}{q}\right) = \frac{1}{p} \rightarrow \left(\frac{pa}{q}\right)^{-\frac{1}{p}} \times \frac{1}{p} a = \frac{1}{p}$$

$$\frac{pa^{\frac{1}{p}}}{q^{\frac{1}{p}}} \times \frac{a^{\frac{1}{p}}}{q^{\frac{1}{p}}} = \frac{1}{p} \rightarrow a = \left(\frac{q}{p}\right)^{\frac{1}{p}}$$

نقطه مسطح $a = \frac{pa}{q}$ $u=0$ $u = \frac{pa}{q}$

$$x|x| - x \quad \begin{cases} x \geq 0 \rightarrow x^2 - x \\ x < 0 \rightarrow -x^2 - x \end{cases}$$



$$x|x| \geq x \rightarrow D f = [0, +\infty) \cup (-\infty, -1]$$

نقطه بحرانی

$$x \geq 1 \rightarrow f(x) = \sqrt{x^2 - x} \rightarrow f'(x) = \frac{2x-1}{2\sqrt{x^2-x}}$$

x	1	+	+	+
f'	0	-	-	-

نقطه بحرانی

x	1	+	-	+
f'	0	-	-	-

نقطه بحرانی

$$-1 \leq x < 0 \rightarrow f(x) = \sqrt{-x^2 - x} \rightarrow f'(x) = \frac{-2x-1}{2\sqrt{-x^2-x}}$$

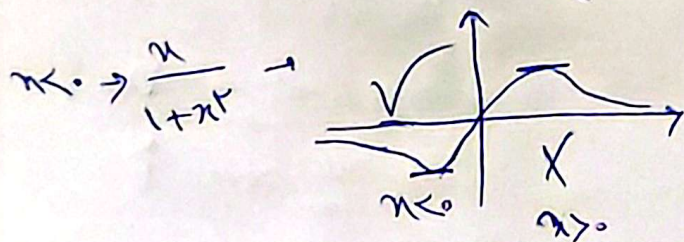
$$f = \frac{m(m-1)-r}{(m-1+m)^2} = \frac{m^2 - m - r}{(2m-1)^2} \rightarrow m^2 - m - r \leq 0$$

$$-1 \leq m \leq r \rightarrow m \neq r + m = (-1, 0]$$

$$x \geq 0 \rightarrow \frac{x}{1-x^2} \rightarrow f'(x) = \frac{1-x^2 - x(2x)}{(1-x^2)^2} = \frac{1-x^2-2x^2}{(1-x^2)^2} = \frac{-x^2+1}{(1-x^2)^2}$$

$$-x^2+1=0 \rightarrow x = \pm \frac{\sqrt{1-x^2}}{x}$$

$$\rightarrow x = \pm \frac{1}{x}$$



نقطه بحرانی $f'(x) = 0$

$$x = \frac{\sqrt{1-x^2}}{x}$$

نقطه بحرانی - تکلیف