پرسمان ساری - دوازدهم تجربی　　　بنام خدا　　　۱۴ آفرین　　　تکلیف سوال ۲۴۰

۱)
$m_{AB} = \frac{5-1}{3-0} = \frac{4}{3} \to \cancel{f'(x)} = \frac{4}{3}$

2

۲)
$A(-1,1), B(3,2) \to m_{AB} = \frac{2-1}{3+1} = \frac{1}{4} \to y-2 = \frac{1}{4}(x-3) \to y = \frac{1}{4}x + \frac{5}{4}$

$\sqrt{ax-1} = \frac{1}{4}x + \frac{5}{4} \to 4\sqrt{ax-1} = x+5$

$x^2 + (8-9a)x + 25 = 0 \to (8-9a)^2 - 100 = 0 \to (8-9a) = \pm 10$

$8-9a = 10 \to a = -\frac{2}{9}$ ✗ , $8-9a = -10 \to a = 2$ ✓

$f(x) = \sqrt{2x-1} \to f(5) = \sqrt{9} = 3$ ✓

2

۳)
$y = \frac{x^2+mx+1}{x+3} \to y' = \frac{(2x+m)(x+3) - (x^2+mx+1)}{(x+3)^2} = y'(1) = \frac{3}{4} \to 4y - 3x = n$

$y = \frac{3}{4}x + \frac{n}{4} \to 4(m+4) - 2 - m \to m = 3 \to \frac{3}{4}x + \frac{n}{4} = y \to \frac{3}{4} + \frac{n}{4} = 1 \to \frac{3+n}{4} = \frac{4}{4}$

$n = 1 \to 1+2 = 3$ ✓

2

۴)
$g(x) = \frac{3}{3+\sin x} \to g'(x) = \frac{-3\cos x}{(3+\sin x)^2} \to g'(\frac{5\pi}{6}) = \frac{-6}{(6-\sqrt{3})^2}$

$f(x) = \frac{27 - \sin^3 x}{9 - \sin^2 x} = \frac{(3-\sin x)(9 + \sin^2 x + 3\sin x)}{(3+\sin x)(3-\sin x)} = \frac{\sin^2 x + 3\sin x + 9}{3 + \sin x}$

$f'(x) = \frac{(2\sin x\cos x + 3\cos x)(3+\sin x) - (\cos x)(\sin^2 x + 3\sin x + 9)}{(3+\sin x)^2}$

1,5

$f'(\frac{5\pi}{6}) = \frac{\frac{3\sqrt{3}}{2} + 2\sqrt{3}}{(6-\sqrt{3})^2} \to 3g'(\frac{5\pi}{6}) - f'(\frac{5\pi}{3}) = \frac{1+2\sqrt{3}}{22}$

۵)
$g'(x) \times f'(g(x)) = (f(g(x)))' \xrightarrow{x=0} g(x) = \frac{1}{\cancel{2}\sqrt{x}} \to f(g(x)) = f(\frac{1}{\cancel{2}\sqrt{x}}) = -x$

$f(x) = -\frac{1}{5\sqrt{2x}}$

2

۱۵/۵ ستاری

۶- $g(x)=\frac{f(x)-1}{x}=\frac{\left(\frac{-1+\sin x}{1+\sin x}\right)^2-1}{x}=\frac{-4\sin x}{x(1+\sin x)^2}$; $\sin x\sim x\to\frac{-4}{x(1+x^2+2x)}$

$-\frac{4}{1}=-4$ ✓ ۲

۷- $f'(\alpha)f'(-\alpha)=-1\to f(x)=-x^2-1\to f'(x)=-2x$

$(-2\alpha)(2\alpha)=-1\to\alpha^2=\frac{1}{4}$, $f(\alpha)=f(-\alpha)=-\alpha^2-1=-\frac{5}{4}\to -1/25$ ۲

۸- $f(x)=2\sqrt{x}(4x^2+5)=mx\to f'(x)=20x^{\frac{3}{2}}+5x^{-\frac{1}{2}}=m\to 10x^{\frac{5}{2}}+5m^{\frac{1}{2}}=\left(\sqrt{10x^5}+\frac{1}{\sqrt{5x^2}}\right)x$

$\sqrt{x^5}+\sqrt{4x}=\sqrt{2x^5}+\sqrt{5m}\to\sqrt{12x^5}-\sqrt{5x}=0\to 3\sqrt{x}(4x^2-1)=0\to x=0$ ✗ , $x=\pm\frac{1}{2}$ ✓

$x=-\frac{1}{2}$ ✗

$m=20\sqrt{\left(\frac{1}{2}\right)^3}+5\sqrt{2}=8\sqrt{2}$ ✓ ۲

۹- $\sqrt{x}=t\to f(x)=\frac{t}{-2t+5t^2+1}\to\frac{2}{-2+5t+t^2+1}=at^2\to-2at^3+5at^4+at^2-1=0$

$-10at^4+12at^2+a=0\to a(5t^4-3t^2-1)=0\to\left(t^2-\frac{1}{2}\right)\left(t^2+\frac{1}{5}\right)\to t^2=\frac{1}{2}$ ✓

۱/۵ $t^2=-\frac{1}{5}$ ✗

۱۰- $f(g(x))'=g'(x)\times f'(g(x))\to g(x)=\frac{1}{\sqrt{x^2-1}}=(x^2-1)^{-\frac{1}{2}}\to g'(x)=-\frac{1}{2}(x^2-1)^{-\frac{3}{2}}\times 2x$

$g'\left(\sqrt{\frac{5}{4}}\right)=-4\sqrt{5}\to x\to\sqrt{\frac{5}{4}}\to g(x)=\frac{1}{\sqrt{\frac{1}{4}}}=2^+\to x\to 2^+=[2^+]=2\to f(x)=(2x)^3=8x^3$

$f'(x)=24x^2-f'(2^+)=96\to f'\left(g\left(\sqrt{\frac{5}{4}}\right)\right)\times g'\left(\sqrt{\frac{5}{4}}\right)=96\times 4\sqrt{5}\to$ ۸ برابر ✓ ۲

۴

$$3g - f(x) = \frac{9}{3+\sin x} - \frac{(\cancel{3-\sin x})(9+\sin^2 x+3\sin x)}{(\cancel{3-\sin x})(3+\sin x)} = \frac{-\sin x(\cancel{\sin x+3})}{\cancel{\sin x+3}}$$

یا $-\sin x \xrightarrow{\text{مشتق}} (3g-f)'(x) = -\cos x \rightsquigarrow -\cos(\frac{5\pi}{3}) = \boxed{-\frac{1}{2}}$

۹

$$y = mx \longrightarrow \frac{\sqrt{a}}{-2a^2+a+1} = ma \longrightarrow \frac{1}{-2a^2+a+1} = m\sqrt{a}$$

$$m\sqrt{a}(-2a^2+a+1) = 1 \longrightarrow -2m(a^{\frac{5}{2}}) + m(a^{\frac{3}{2}}) + m(a)^{\frac{1}{2}} = 1 \quad \xrightarrow{\text{مشتق}}$$

$$-5m(a^{\frac{3}{2}}) + \frac{3}{2}m(a^{\frac{1}{2}}) + \frac{m}{2}(a^{-\frac{1}{2}}) = 0$$

$$\frac{m}{2}(a^{-\frac{1}{2}})(-10a^2+3a+1) = 0 \longrightarrow a = -\frac{1}{5} \text{ یا } a = \frac{1}{2} \ (a>0)$$

$$f(a) = \frac{\frac{\sqrt{2}}{2}}{-2(\frac{1}{4})+\frac{1}{2}+1} = \frac{\frac{\sqrt{2}}{2}}{1} = \boxed{\frac{\sqrt{2}}{2}}$$