بنام خدا

پویا فلفلیان

DATE / / SUBJECT:

۱)

$$g'(x)=f'(\tan^2 x+\sqrt{2}\cos x)\times\left(2\tan x\times(1+\tan^2 x)+(-\sqrt{2})\sin x\right)\rightarrow$$

$$g'(\tfrac{\pi}{4})=f'(2)\times\left(4+(-\sqrt{2})\times\tfrac{\sqrt{2}}{2}\right)=3f'(2)=\sqrt{3}\rightarrow \boxed{f'(2)=\frac{\sqrt{3}}{3}}$$

۲)

$$f(x)=\frac{(2+\cos x)(\cos^2 x-2\cos x+4)}{(2+\cos x)(2-\cos x)}=\frac{\cos^2 x-2\cos x+4}{2-\cos x}$$

$$\rightarrow f(x)-2g(x)=\frac{\cos^2 x-2\cos x}{2-\cos x}=\frac{\cos x(\cos x-2)}{2-\cos x}=-\cos x\rightarrow (f(x)-2g(x))'=$$

$$f'(x)-2g'(x)=\sin x\rightarrow f'(\tfrac{7\pi}{6})-2g'(\tfrac{7\pi}{6})=\boxed{\frac{-1}{2}}$$

۳)

$$\lim_{x\to(\frac{3}{4})^+}\frac{4(x-\frac{3}{4})\sqrt{ax}-0}{x-\frac{3}{4}}=4\sqrt{ax}=\frac{4\times\sqrt{3a}}{2}=2\sqrt{3a}$$

$$\lim_{x\to(\frac{3}{4})^-}\frac{-4(x-\frac{3}{4})\sqrt{ax}-0}{x-\frac{3}{4}}=-4\sqrt{ax}=-\frac{4\times\sqrt{3a}}{2}=-2\sqrt{3a}$$

$$\Rightarrow 4\sqrt{3a}=2\sqrt{6}\rightarrow 48a=24\rightarrow \boxed{a=\frac{1}{2}}$$

۴)

$$y=2-ax\rightarrow f(4)=2-4a\ ,\ f'(4)=-a\rightarrow f(4)+f'(4)=2-5a=-1\rightarrow a=\frac{3}{5}$$

$$\rightarrow f'(4)=-a=\boxed{\frac{-3}{5}}$$

پویا فلاحیان

DATE / / SUBJECT:

۵

$y=\frac{1}{7}(x+5) \rightarrow$ شیب $=\frac{1}{7} \rightarrow \frac{a+3}{(3x+1)^2}=\frac{1}{7}$ (1) , $\frac{1}{7}(x+5)=\frac{ax-1}{3x+1}$ (2) $\rightarrow$

(2) $\frac{1}{7}(3x^2+16x+5)=ax-1 \rightarrow \frac{1}{7x}(3x^2+16x+12)=a \xrightarrow{(1)} \dfrac{\frac{1}{7x}(3x^2+16x+12)+3}{(3x+1)^2}=\frac{1}{7}$

$\rightarrow \frac{3x^2+37x+12}{7x(3x+1)^2}=\frac{1}{7} \rightarrow 9x^3+6x^2+x=3x^2+37x+12 \rightarrow 9x^3+3x^2-36x-12=0$

$\rightarrow 3x^3+x^2-12x-4=0 \xrightarrow{x>0} x=2 \rightarrow \frac{a+3}{7\times 7}=\frac{1}{7} \rightarrow \boxed{a=4}$

۶

مماس بر محور xها $\rightarrow 0=$ شیب $\rightarrow f'(2)=3x^2+a=12+a=0 \rightarrow \boxed{a=-12}$, $f(2)=0 \rightarrow 8+2a-b=0$

$\rightarrow \boxed{b=-16} \rightarrow b-a=-16+12=\boxed{-4}$

۷

$\lim\limits_{x\to 0}\frac{f(x)\,f'(x)}{(1-\cos x)^m}=\frac{0}{0}$

$\frac{\sin^n(x^2)\times n\sin^{n-1}(x^2)\times 2x\times\cos x^2}{(1-\cos x)^m}=\frac{n\,x^{2n}\times x^{2n-2}\times 2x}{\left(\frac{x^2}{2}\right)^m}=\frac{2n\,x^{4n-1}\times 2^m}{x^{2m}} \rightarrow$

$n2^{m+1}\times x^{4n-1-2m}=2^{\frac{n}{2}} \rightarrow 4n-1=2m \rightarrow \frac{2m+1}{4}\times 2^{m+1}=2^{\frac{11}{2}} \rightarrow \boxed{m=\frac{7}{2}} \rightarrow \boxed{n=2}$

$\rightarrow 2m+n=7+2=\boxed{9}$

۸

$y=\frac{\sqrt{x}+1}{\sqrt{x}-1} \rightarrow \frac{y+1}{y-1}=\sqrt{x} \rightarrow x=\left(\frac{y+1}{y-1}\right)^2 \rightarrow y=\left(\frac{x+1}{x-1}\right)^2 \rightarrow {f^{-1}}'(2)=$

$2\left(\frac{3}{1}\right)\times\frac{-2}{1}=\boxed{-12}$

Sahand

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DATE / / SUBJECT:

۹

$f(-1) = ۸(-a+1)$, $f(0) = ۱ \rightarrow \frac{۱+۸a-۸}{۱} = ۸a-۷ = -۱۱ \rightarrow \boxed{a = \frac{-۱}{۲}} \rightarrow$

$h = -۲a = ۱$

$f'(\frac{۱}{۲}) = ۳ \times ۴ \times ۲ \times \frac{۱}{۲} + (\frac{-۱}{۲})(۸) = ۱۲ - ۴ = \boxed{۸}$

۱۰

$\sin\frac{\pi}{۲} \times \cos\frac{\pi}{۲} = 0$, $\sin\frac{\pi}{۲} \times \cos\pi = -۱ \rightarrow \frac{-۱}{\frac{\pi}{۴}} = \frac{-۴}{\pi}$

$y = \sin^۲ x - \cos^۲ x = -\cos ۲x \rightarrow -\cos\frac{\pi}{۲} = 0$, $-\cos\pi = ۱ \rightarrow \frac{۱}{\frac{\pi}{۴}} = \frac{۴}{\pi}$

$\frac{\frac{-۴}{\pi}}{\frac{۴}{\pi}} = \boxed{-۱}$