۱)

$$g'(x) = \left(2\tan x(1+\tan^2 x) - \sqrt{2}\sin x\right) \times f'(\tan^2 x + \sqrt{2}\cos x)$$

$$g'\left(\frac{\pi}{4}\right) = \left(\underbrace{2(1)(2) - 1}_{3}\right) \times f'(2) = \sqrt{3} \to f'(2) = \frac{\sqrt{3}}{3}$$

---

۲)

$$f(x) = \frac{\cancel{(2+\cos x)}(\cos^2 x - 2\cos x + 4)}{\cancel{(2+\cos x)}(2-\cos x)} \to \frac{f}{g}(x) = \frac{\cos^2 x - 2\cos x}{2-\cos x} = -\cos x$$

$$\left(\frac{f}{g}\right)'(x) = \sin x \xrightarrow{x = \frac{7\pi}{6}} \sin\frac{7\pi}{6} = -\frac{1}{2}$$

---

۳)

$$f'_+\left(\frac{3}{4}\right) = \lim_{x \to \frac{3}{4}^+} \frac{f(x) - f(\frac{3}{4})}{x - \frac{3}{4}} = \frac{4\cancel{(x-\frac{3}{4})}\sqrt{ax} - 0}{\cancel{x-\frac{3}{4}}} = 4\sqrt{ax}$$

$$f'_-\left(\frac{3}{4}\right) = \lim_{x \to \frac{3}{4}^-} \frac{f(x) - f(\frac{3}{4})}{x - \frac{3}{4}} = \frac{(3-4x)\sqrt{ax}}{x - \frac{3}{4}} = -4\sqrt{ax}$$

$$8\sqrt{ax} = 4\sqrt{6} \to 4ax = 6 \to a\cdot\frac{3}{4} = \frac{6}{4} \to a = 2$$

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۴)

$$f'(4) = -a$$
$$f(4) = -4a + 2 \longrightarrow -5a + 2 = -1 \to a = \frac{3}{5} \to f'(4) = -\frac{3}{5}$$

---

۵)

$$y = \frac{x+5}{7} \qquad \frac{x+5}{7} = \frac{ax-1}{3x+1} \longrightarrow 3x^2 + 16x + 5 = 7ax - 7$$

$$3x^2 + (16 - 7a)x + 12 = 0$$

$$(16 - 7a)^2 = 4 \times 3 \times 12 \to 16 - 7a = 12 \to a = \frac{4}{7}$$
$$16 - 7a = -12 \to a = 4$$

$$a = 4 \to 3x^2 - 12x + 12 = 3(x^2 - 4x + 4) = 0$$
$$x = 2 \checkmark$$

۶)

$f'(2)=0 \to 3x^2+a \to a=-12$

$f(2)=0 \to 8-12(2)-b=0 \to b=-16 \to -16+12=-4$

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۷)

$x\to 0 \implies f(x)\sim x^{2n}$

$f'(x)\sim 2n x^{2n-1}$ $\qquad n=2,\ m=\frac{3}{2} \to 2m+n=5$

$\dfrac{2n x^{4n-1}}{\left(\frac{x^2}{2}\right)^m}=32\sqrt{2} \to \lim\limits_{x\to 0} x^{4n-2m-1}\times 2^{m+1}\times n=32\sqrt{2}$ $\to 2^{2n+\frac{1}{2}}\times n=2^{\frac{11}{2}}$

آن توان x صفر شود در نهایت تا $32\sqrt{2}$ شود

$2m=4n-1 \to m=2n-\frac{1}{2}$

$\text{if } n=2 \to 2^{\frac{9}{2}}\times 2^1=2^{\frac{11}{2}}$ ✓

n حتما باید توان کوچکی از ۲ باشد وگرنه تساوی برقرار نبود و فقط n=۲ جواب است.

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۸)

$(f^{-1})'(2)$

$f(x)=2 \to \dfrac{\sqrt{x}+1}{\sqrt{x}-1}=2 \to \sqrt{x}+1=2\sqrt{x}-2 \to \sqrt{x}=3 \to x=9$

$f'(x)=\dfrac{1}{2\sqrt{x}}\times\dfrac{-2}{(\sqrt{x}-1)^2}\Big|_{x=9} \to \dfrac{1}{6}\times\dfrac{-2}{4}=-\dfrac{1}{12} \to (f^{-1})'(2)=-12$

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۹)

$\dfrac{f(0)-f(-1)}{0+1}=-11 \to \underset{1}{f(0)}-f(-1)=-11 \to f(-1)=12$

$8(-a+1)=12 \to -a+1=\frac{3}{2} \to a=-\frac{1}{2}$ $\qquad x=-2a=1$

$3(x^2+1)^2(2x)(-\frac{1}{2}x+1)+(x^2+1)^3(-\frac{1}{2})$

$3(1+1)^2\times 2(1)(\frac{1}{2})+(8)(-\frac{1}{2})$ $\qquad -4$

$3\times 4+-4=8$

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۱۰)

$y=-\cos 2x=g(x)$ $\qquad \xrightarrow{\text{آهنگ متوسط}} \dfrac{f(\frac{\pi}{2})-f(\frac{\pi}{4})}{g(\frac{\pi}{2})-g(\frac{\pi}{4})}=\dfrac{1(-1)-0}{1-0}=-1$

$f(x)=\sin x\cos 2x$

نوید امیری - تکلیف ۲۳