

نام و نام خانوادگی شماره کلاس پاسخنامه تشریحی تکلیف شماره ۲۳ *پاسخنامه تشریحی تکلیف شماره ۲۳*

$$g^{(*)} = f(\tan^2 u + \sqrt{2} \cos u) \rightarrow g'(u) = (\tan^2 u + \sqrt{2} \cos u) \left(f'(\tan^2 u + \sqrt{2} \cos u) \right) = (2 \times (1 + \tan^2 u)(\tan u) - \sqrt{2} \sin u) (f'(\tan^2 u + \sqrt{2} \cos u))$$

$$g'\left(\frac{\pi}{2}\right) = \sqrt{3} = (2(1 + \tan^2 \frac{\pi}{2})(\tan \frac{\pi}{2}) - \sqrt{2} \sin \frac{\pi}{2}) \times f'\left(\tan^2 \frac{\pi}{2} + \sqrt{2} \cos \frac{\pi}{2}\right) \sim (2-1) f'(2) = \sqrt{3} \rightarrow \frac{\sqrt{3}}{2} = f'(2)$$

$$f(u) = \frac{1 + \cos^2 u}{\sin^2 u} = \frac{(1 + \cos u)(1 - \cos u + \cos^2 u)}{(1 + \cos u)(1 - \cos u)} = \frac{1 - \cos u + \cos^2 u}{1 - \cos u}$$

$$g(u) = \frac{u}{1 - \cos u} \rightarrow f(u) - u g(u) = \frac{1 - \cos u + \cos^2 u - u}{1 - \cos u} = \frac{\cos^2 u - \cos u}{1 - \cos u} = \frac{\cos u (\cos u - 1)}{(1 - \cos u)} = -\cos u \xrightarrow{\text{مشتق}} \sin(u)$$

$$\rightarrow \sin \frac{\pi}{2} = \frac{1}{1} = 1$$

$$|x_2 - 1| \sqrt{a} \rightarrow f(u) = \pm (\sin u - 1) (\sqrt{a} u) \xrightarrow{f'(\frac{\pi}{2})} \pm \sin u = \pm \sqrt{3} a \sim \text{افتلاف} = \sin \sqrt{3} a = \sqrt{3}$$

$$\rightarrow \sin \sqrt{3} a = 1$$

$$a = \frac{1}{\sqrt{3}}$$

$$\text{مشتق} \rightarrow f'(x) = -a \rightarrow f'(x) = -\frac{1}{8}$$

$$\text{مشتق} \rightarrow f(x) = -ax + 2$$

$$f(x) + f(x) = -2ax - a + 2 \rightarrow -2ax - a + 2 = -1 \rightarrow a = \frac{3}{8}$$

$$\frac{ax - 1}{x^2 + 1} = \frac{x + 2}{x} \rightarrow Vax - 1 = x^2 + 14x + 2 \rightarrow x^2 + (14 - Va)x + 12 = 0 \xrightarrow{\Delta = 0} A^2 - 14 \times 12 = 0$$

$$\rightarrow A = \pm 12 \rightarrow \begin{matrix} x < 0 & \leftarrow & 0 < A \\ x > 0 & \leftarrow & 0 > A \end{matrix}$$

$$\rightarrow 14 - Va = -12 \rightarrow Va = 26 \rightarrow a = \frac{13}{13}$$

$$f(x) = x^2 + ax - b \rightarrow f'(x) = 0 \rightarrow 2x + a = 17 + a = 0 \rightarrow a = -17$$

$$\rightarrow f(x) = 0 \rightarrow 1 + 2a - b = 0 \rightarrow 1 - 2(-17) - b = 0 \rightarrow b = -14$$

$$b - a = -14 + 17 = 3$$

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$$f(x) = \sin^n(x) \rightarrow f'(x) = n \sin^{n-1}(x) \cos(x) \rightarrow \frac{f'(x)}{f(x)} = \frac{n \sin^{n-1}(x) \cos(x)}{\sin^n(x)} = \frac{n \cos(x)}{\sin(x)} = n \cot(x)$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \cot(x) = 0$$

$$\frac{n \cos(x)}{\sin(x)} = n \cot(x) \rightarrow \lim_{x \rightarrow \frac{\pi}{2}} n \cot(x) = n \cdot 0 = 0$$

$$\rightarrow \frac{n \cos(x)}{\sin(x)} = n \cot(x) \rightarrow \lim_{x \rightarrow \frac{\pi}{2}} n \cot(x) = n \cdot 0 = 0$$

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$$\frac{\sqrt{x+1}}{\sqrt{x-1}} = y \rightarrow \sqrt{x+1} = y \sqrt{x-1} \rightarrow x+1 = y^2(x-1) \rightarrow x+1 = y^2x - y^2 \rightarrow x(1-y^2) = -y^2 - 1 \rightarrow x = \frac{-y^2-1}{1-y^2}$$

$$\rightarrow \frac{d}{dx} \left(\frac{\sqrt{x+1}}{\sqrt{x-1}} \right) = \frac{y}{x} = \frac{y}{\frac{-y^2-1}{1-y^2}} = \frac{y(1-y^2)}{-y^2-1} = \frac{y(1-y^2)}{-(y^2+1)} = -\frac{y(1-y^2)}{y^2+1}$$

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* سوال: اگر $f(x) = \frac{1}{x}$ و $f'(x) = -\frac{1}{x^2}$ باشد، آنگاه $f'(1) = -1$ صحیح است یا نه؟

$$f(x) = (x+1)^2(x-1) \rightarrow f'(x) = 2(x+1)(x-1) + (x+1)^2 = 2(x^2-1) + (x^2+2x+1) = 3x^2+2x-1$$

$$f'(1) = 3(1)^2+2(1)-1 = 3+2-1 = 4$$

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$$y = \sin x \cos x \quad \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \rightarrow f\left(\frac{\pi}{2}\right) = 1 \cdot 0 = 0 \rightarrow f\left(\frac{3\pi}{2}\right) = (-1) \cdot 0 = 0$$

$$\rightarrow \frac{f\left(\frac{3\pi}{2}\right) - f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2} - \frac{\pi}{2}} = \frac{0 - 0}{\pi} = 0$$

$$y = \sin^2 x - \cos^2 x \quad \left[\frac{\pi}{2}, \frac{3\pi}{2} \right] \rightarrow f\left(\frac{\pi}{2}\right) = 1 - 0 = 1 \rightarrow f\left(\frac{3\pi}{2}\right) = 0 - 1 = -1$$

$$\rightarrow \frac{f\left(\frac{3\pi}{2}\right) - f\left(\frac{\pi}{2}\right)}{\frac{3\pi}{2} - \frac{\pi}{2}} = \frac{-1 - 1}{\pi} = -\frac{2}{\pi}$$

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