نام و نام خانوادگی امیرعلی تنهایی راد .... پاسخنامه تشریحی تکلیف شماره ۲۳ ...... کلاس دوازدهم ریاضی B ....

۱۵/۲۵

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$g'(x) = f'(\tan^2 x + \sqrt{2}\cos x)\,((2\tan x)(1+\tan^2 x))(-\sqrt{2}\sin x)$

$g'(\frac{\pi}{4}) = f'(1^2 + \sqrt{2}\times\frac{\sqrt{2}}{2})\,((2\times 1)(1+1))(-\sqrt{2}\times\frac{\sqrt{2}}{2}) \Rightarrow$

$g'(\frac{\pi}{4}) = f'(2)\times 4\times -1 \to \sqrt{3} = f'(2)\times -4 \to f'(2) = \frac{-\sqrt{3}}{4}$

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$f(x) = \frac{(2+\cos x)(4+\cos^2 x - 2\cos x)}{(2-\cos x)(2+\cos x)} = \frac{\cos^2 x - 2\cos x + 4}{2-\cos x}$ $\quad 2g(x) = \frac{4}{2-\cos x}$

$f'(\frac{7\pi}{6}) - 2g'(\frac{7\pi}{6}) = (f-2g)'(\frac{7\pi}{6})$ $\quad f(x) - 2g(x) = \frac{\cos^2 x - 2\cos x}{2-\cos x} = \frac{\cos x(\cos x - 2)}{2-\cos x} =$

$-\cos x \longrightarrow (f-2g)' = \sin x \longrightarrow (f-2g)'(\frac{7\pi}{6}) = \sin\frac{7\pi}{6} = \frac{-1}{2}$ ✓

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$f(x) = \begin{cases} (4x-3)(\sqrt{ax}) & x > \frac{3}{4} \\ (-4x+3)(\sqrt{ax}) & x < \frac{3}{4} \end{cases}$ $\quad f'(x) = \begin{cases} (4\sqrt{ax}) + (\frac{a}{2\sqrt{ax}}\times(4x-3)) & \frac{3}{4} \\ (-4\sqrt{ax}) + (\frac{a}{2\sqrt{ax}}\times(-4x+3)) & x < \frac{3}{4} \end{cases}$

$\longrightarrow 8\sqrt{ax} + \frac{8ax - 6a}{2\sqrt{ax}} = \frac{\overset{24ax-6a}{\cancel{16ax + 8ax - 6a}}}{2\sqrt{ax}} = \frac{6a(4x-1)}{2\sqrt{ax}} = \frac{6a(2)}{2\sqrt{ax}} = \frac{12a}{\sqrt{3a}}$

$\frac{12a}{\sqrt{3a}} = 2\sqrt{6} \longrightarrow \frac{6a}{\sqrt{3a}} = \sqrt{6} \longrightarrow 6a = \sqrt{18a} \longrightarrow 36a^2 = 18a \longrightarrow 2a^2 = a \longrightarrow 2a = 1 \longrightarrow a = \frac{1}{2}$

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$y = -ax + 2 \longrightarrow f'(4) = -a \qquad f(4) = -4a + 2$

$f(4) + f'(4) = -5a + 2 = -1 \longrightarrow 5a = 3 \longrightarrow a = \frac{3}{5}$

$f'(4) = -a = -\frac{3}{5}$ ✓

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$\frac{an-1}{3n+1} = \frac{n}{7} + \frac{5}{7} \longrightarrow 7an - 7 = 3n^2 + n + 15n + 5$

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$3n^2 + n(16-7a) + 12 = 0 \xrightarrow{\Delta = 0} (16-7a)^2 - 12^2 = \underbrace{(16-7a-12)}_{a=\frac{4}{7}}\underbrace{(16-7a+12)}_{a=4}$

$a = 4 \longrightarrow 3n^2 - 12n + 12 \longrightarrow n = 2$ ✓

$a = \frac{4}{7} \longrightarrow 3n^2 + 12n + 12 \longrightarrow n = -2$ نامقبول x

$$f(2)=0 \longrightarrow 8+2a-b=0$$

$$f'(x)=3x^2+a \longrightarrow f'(2)=0 \longrightarrow 12+a=0 \longrightarrow a=-12$$

$$\Longrightarrow 8+2(-12)-b=0 \longrightarrow b=-16$$

$$b-a=-16-(-12)=-4$$

2 — 6

$$\lim_{x\to 0}\frac{\sin^n(x^2)\times 2nx\sin^{n-1}(x^2)\times\cos x^2}{\left(2\sin^2\frac{x}{2}\right)^m}\xrightarrow{\text{هم‌ارزی}}\lim_{x\to 0}\frac{(x^2)^n\times n\times(x^2)^{n-1}\times 2x}{\left(2\left(\frac{x}{2}\right)^2\right)^m}=\frac{(x)^{2n+2n-2+1}\times 2n}{(x)^{2m}\times\frac{1}{2^m}}$$

0 — 7

$$=\frac{2nx^{4n-1}}{\frac{1}{2^m}\times x^{2m}}=2^{m+1}\times n\times x^{4n-2m-1}\Rightarrow 4n-2m-1=0\longrightarrow n=\frac{2m+1}{4}$$

$$\hookrightarrow 2^{m+1}\times\frac{2m+1}{4}=32\sqrt{2}\longrightarrow m=\frac{7}{2},\ n=2\longrightarrow 2m+n=9$$

$$f'(x)=\frac{\frac{1}{2\sqrt{x}}(\sqrt{x}-1)-\frac{1}{2\sqrt{x}}(\sqrt{x}+1)}{(\sqrt{x}-1)^2}=\frac{\frac{1}{2}-\frac{1}{2\sqrt{x}}-\frac{1}{2}-\frac{1}{2\sqrt{x}}}{(\sqrt{x}-1)^2}=\frac{\frac{-2}{2\sqrt{x}}}{(\sqrt{x}-1)^2}=\frac{\frac{-1}{\sqrt{x}}}{(\sqrt{x}-1)^2}$$

$$f'(2)=\frac{\frac{-1}{\sqrt{2}}}{(\sqrt{2}-1)^2}=\frac{\frac{-\sqrt{2}}{2}}{3-2\sqrt{2}}\times\frac{3+2\sqrt{2}}{3+2\sqrt{2}}=\frac{\frac{-3\sqrt{2}}{2}-2}{1}=\frac{-3\sqrt{2}}{2}-\frac{4}{2}$$

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$$=\frac{-3\sqrt{2}-4}{2}\xrightarrow{\text{معکوس}}\frac{-2}{3\sqrt{2}+4}\times\frac{3\sqrt{2}-4}{3\sqrt{2}-4}=\frac{-6\sqrt{2}+8}{2}=-3\sqrt{2}+4$$

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$$f(0)=(0+1)^3(1)=1$$

$$f(-1)=(+2)^3(-a+1)=-8a+8\longrightarrow\frac{1-(-8a+8)}{1}=\frac{8a-7}{1}=-11$$

2 — 9

$$8a=-4\rightarrow a=\frac{-1}{2}\longrightarrow -2a=1$$

$$f'(x)=3(x^2+1)^2(2x)(ax+1)+a(x^2+1)^3\Rightarrow f'(1)=\left(3\times4\times2\times\frac{1}{2}\right)+\left(\frac{-1}{2}\times8\right)$$

$$=12+(-4)=8$$

$$y=\sin x\cos 2x\ \begin{cases}\xrightarrow{x=\frac{\pi}{4}} y=\frac{\sqrt{2}}{2}\times 0=0\\ \xrightarrow{x=\frac{\pi}{2}} y=1\times -1=-1\end{cases}\longrightarrow\frac{-1}{\frac{\pi}{2}-\frac{\pi}{4}}$$

2 — 10

$$y=(\sin^2x-\cos^2x)(\sin^2x+\cos^2x)=-\cos 2x\ \begin{cases}\xrightarrow{x=\frac{\pi}{4}} y=0\\ \xrightarrow{x=\frac{\pi}{2}} y=1\end{cases}\longrightarrow\frac{1}{\frac{\pi}{2}-\frac{\pi}{4}}$$

$$\Longrightarrow\frac{\frac{-1}{\cancel{\frac{\pi}{2}-\frac{\pi}{4}}}}{\frac{1}{\cancel{\frac{\pi}{2}-\frac{\pi}{4}}}}=-1$$

$f^{-1}(۲)=t \to f(t)=۲ \to \sqrt{t}+۱ = ۲\sqrt{t}-۲ \to \sqrt{t}=۳ \to t=۹$ ۸

$(f^{-1})'(۲) = \frac{۱}{f'(۹)}$, $f'(x)=\frac{-۲}{(\sqrt{x}-۱)^{۲}}\times\frac{۱}{۲\sqrt{x}}$

$f'(۹)=\frac{-۲}{(۲)^{۲}}\times\frac{۱}{۶}=-\frac{۱}{۱۲} \to \boxed{(f^{-1})'(۲)=-۱۲}$

$g(x)=f'(\tan^{۲}x+\sqrt{۲}\cos x)\times(۲\tan x(۱+\tan^{۲}x)-\sqrt{۲}\sin x)$ ۱

$g'(\frac{\pi}{۴})=f'(۱+\sqrt{۲}\times\frac{\sqrt{۲}}{۲})\times ۲\times ۱\times(۱+۱^{۲})-(\sqrt{۲}\times\frac{\sqrt{۲}}{۲})$

$\underset{\sqrt{۳}}{\cancel{g'(\frac{\pi}{۴})}} = f'(۲)\times[\underset{۳}{\cancel{۲(۲)}}-۱] \to f'(۲)=\frac{\sqrt{۳}}{۳}$